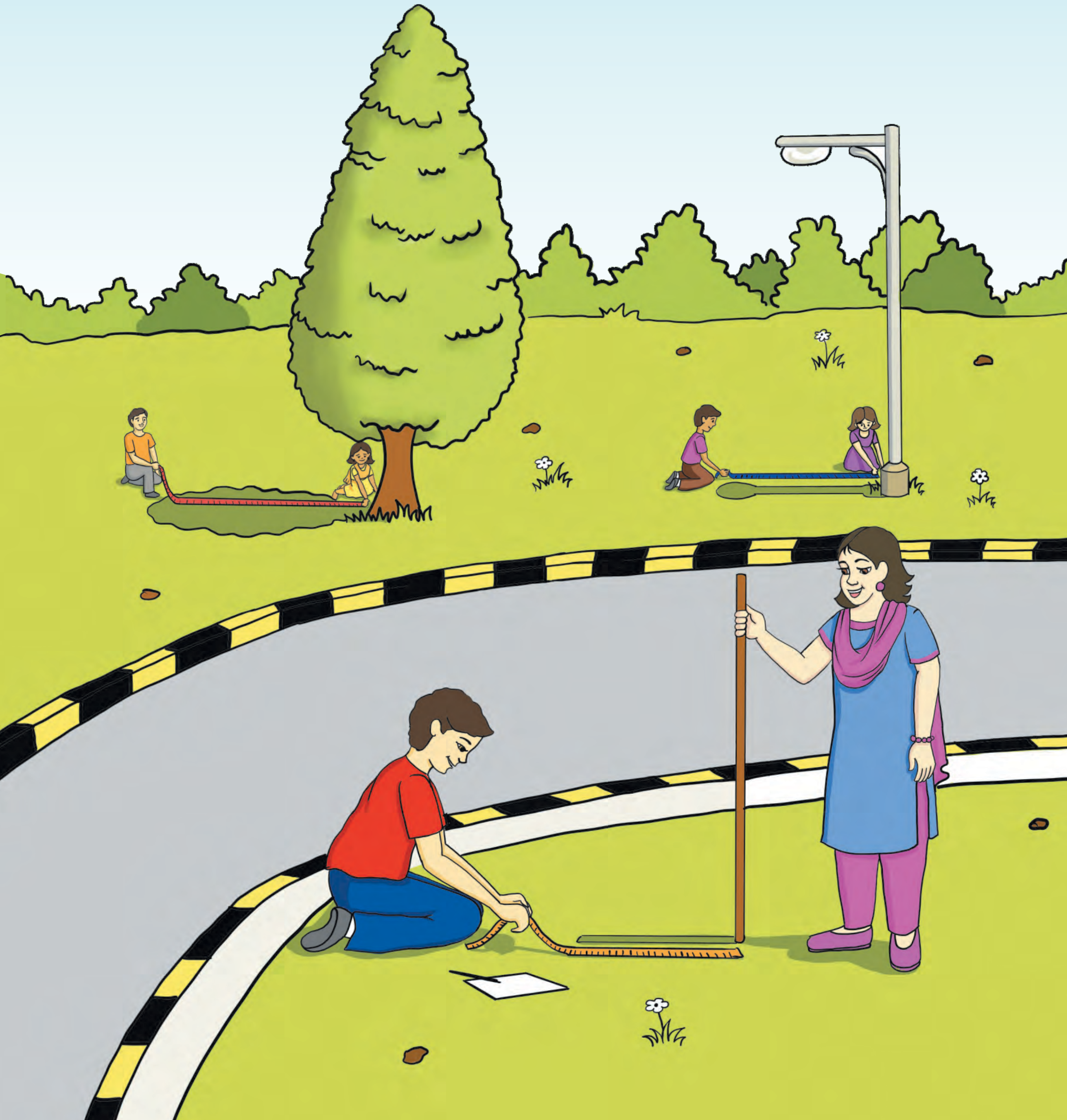
MATHEMATICS

Part - II

STANDARD NINE



The coordination committee formed by GR No. Abhyas - 2116/(Pra.Kra.43/16) SD - 4
Dated 25.4.2016 has given approval to prescribe this textbook in its meeting held on 3.3.2017

MATHEMATICS

Part-II

STANDARD NINE

Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune.

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First Edition : 2017
Reprint : 2020

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Typesetting : D.T.P Section
Textbook Bureau, Pune.

Paper : 70 GSM Cream wove

Printer : SHRI GANRAJ PRINTING AND BINDING WORKS, KOLHAPUR

Print Order No. : N/PB/2020-21/75,000

Publisher

Vivek Uttam Gosavi, Controller
Maharashtra State Textbook Bureau,
Prabhadevi, Mumbai - 400 025.

The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens :

JUSTICE, social, economic and political ;

LIBERTY of thought, expression, belief, faith and worship ;

EQUALITY of status and of opportunity ; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation ;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā
uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders respect, and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone lies my happiness.

Preface

Dear Students,

Welcome to the ninth standard!

You are now going to begin your studies at the secondary level after completing your primary education curriculum. You had only one Mathematics textbook up to the eighth standard, now you will use two textbooks – Mathematics Part-I and Mathematics Part-II.

Up to the eighth standard you have verified the properties of lines, triangles, quadrilaterals, circles, etc. given in the textbook. Now you are going to give logical proofs of these and some more properties. The skill of logical reasoning is of utmost importance in all fields of life. This textbook gives you an opportunity to learn the skill gradually.

Different activities are given in the textbook to help you understand different concepts. Other activities have been provided for revision and additional practice. You are expected to do all these and learn the proofs of properties. Discuss the reason behind every step of a proof and learn the property.

In this textbook, Mathematics-Part II, two new topics namely Trigonometry and Co-ordinate Geometry are introduced. These topics will provide a foundation for higher studies. The study of Surface Area and Volume will be useful in day to day life.

Use of internet will also help you to understand the subject. You will get through the course joyfully if you follow the three point plan of – a deep study of the textbook, activity-based learning and ample practice.

So come on! Let us study Mathematics in the company of our teachers, parents, friends and the internet. Best wishes to you for your studies!

(Dr Sunil Magar)
Director
Maharashtra State Bureau of Textbook
Production and Curriculum Research, Pune.

Pune
Date : 28 April, 2017
Akshaya Tritiya
Indian Solar Year :
8 Vaishakh 1939

It is expected that students will develop the following competencies after studying Mathematics Part II syllabus in Standard IX

Area	Topic	Competency statement
1. **Geometry**		The students will be able to –
	1.1 Euclidean Geometry	• write 'what is given' and 'what is to be proved' from the given statement. • write the proof of the given statements by using logical conclusions.
	1.2 Parallel lines and pairs of angles	• identify the pairs of angles made by a transversals of parallel lines. • understand the properties of pairs of angles and make use of them.
	1.3 Theorems on angles and sides of a triangle.	• write 'Given' 'To prove' and 'proof' of the statements.
	1.4 Similar triangles	• identify similar triangles and write the ratios of corresponding sides.
	1.5 Circle	• prove the properties of chord of circle using tests of congruence of triangles. • draw incircle and circumcircle.
	1.6 Geometric constructions	• construct triangles if different type of information is given.
	1.7 Quadrilateral	• write proofs of the properties of different types of quadrilaterals. • use ICT tools to verify the properties of triangle, quadrilateral and circle.
2. **Co-ordinate Geometry**	2.1 Basics of co-ordinate Geometry	• explain the meaning of co-ordinates of a point in a plane. • describe a point by its co-ordinates. • use ICT tools to find the co-ordinates of a point.
3. **Mensuration**	3.1 Surface area and Volume	• find the surface area and volume of a sphere and a cone.
4. **Trigonome-try**	4.1 Introduction to trigonometry	• tell the different trigonometric ratios using similar triangles and Pythagoras theorem and make use of it.

Instructions for teachers

It is expected that the teachers should go through the textbook of Mathematics Part-II for std IX thoroughly. The book contains many activities and practicals. Try to understand the purpose behind them.

The activities are of two types, (1) to write the proofs and (2) practical verification of properties and theorems. A teacher should make use of discussion, question-answers, group activities etc. to carry out the activities and make the text book more useful. A teacher is also expected to encourage the students to do the activities in the book and help them to invent new ones.

It is more important to write the proofs pursuing logical thinking than doing them by heart. The text book contains a variety of examples to enhance students' logical thinking. Teachers should construct more such examples with the help of students. Examples, which require a little higher thinking ability, are star-marked. Teachers should encourage the students who write proofs logically correct but thinking in a different way.

In the process of evaluation, it is advised to make use of open ended questions and of activity-sheets. Teachers should endeavour to develop such methods of evaluation.

The list of practicals given in the text book should be considered as specimen. Teachers can frame different practicals as well as teaching aids of their own using available material. Different activities given in the text book are included in the practicals. We hope that the evaluation method based on all these will be helpful to develop different competencies for further studies.

List of some practicals (specimen)

(1) To find the distance between two points on a number line.

(2) To verify the properties of angles made by a transversal of parallel lines.

(3) To verify the properties of sides and angles of a triangle using Geometric instruments.

(4) To verify the property of median on hypotenuse of a right angled triangle.

(5) To do the construction of a triangle with given specific conditions.

(6) An activity is given in the book to derive the formula of the surface area of a cone. Using the same activity, derive the formula for the area of a circle which is πr^2.

(7) To draw proportionate map of a room on a graph paper by considering the measurements of the things inside the room.

(8) By drawing X and Y-axes on the school ground, ask students to tell the co-ordinates of a students' positions on the ground.

(9) To find the volume of a cylindrical vessel using formula. Then fill the vessel completely with water and find the volume of the water. Compare both the measurements.

Similar activities can be done for different three dimensional objects.

Index

1 Basic concepts in Geometry

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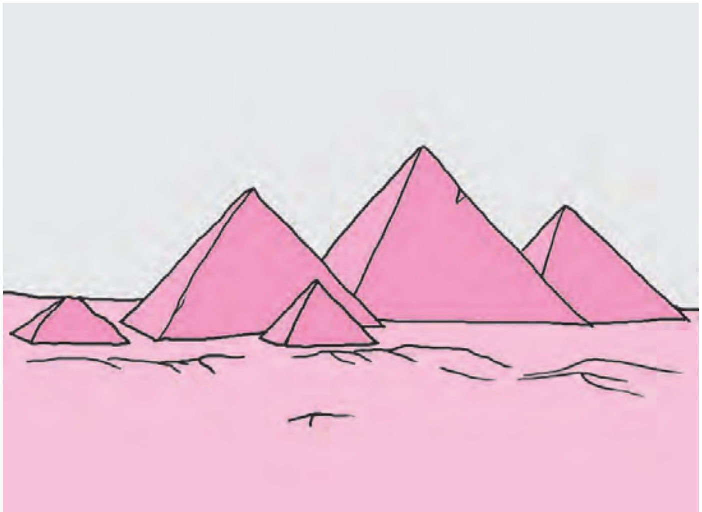
Let's study.

- **Point, line and plane**
- **Co-ordinates of a points and distance**
- **Betweenness**
- **Conditional statements**
- **Proof**



Did you recognise the adjacent picture ? It is a picture of pyramids in Egypt, built 3000 years before Christian Era. How the people were able to build such huge structures in so old time ? It is not possible to build such huge structures without developed knowledge of Geometry and Engineering

The word Geometry itself suggests the origin of the subject. It is generated from the Greek words Geo (Earth) and Metria (measuring). So it can be guessed that the subject must have evolved from the need of measuring the Earth, that is land .

Geometry was developed in many nations in different periods and for different constructions. The first Greek mathematician, Thales, had gone to Egypt. It is said that he determined height of a pyramid by measuring its shadow and using properties of similar triangles.

Ancient Indians also had deep knowledge of Geometry. In vedic period, people used geometrical properties to build altars. The book shulba-sutra describes how to build different shapes by taking measurements with the help of a string. In course of time, the mathematicians Aaryabhat, Varahamihir, Bramhagupta, Bhaskaracharya and many others have given valuable contribution to the subject of Geometry.

Let's learn.

Basic concepts in geometry (Point, Line and Plane)

We do not define numbers. Similarly we do not define a point, line and plane also. These are some basic concepts in Geometry. Lines and planes are sets of points. Keep in mind that the word 'line' is used in the sense 'straight line'.

Co-ordinates of points and distance

Observe the following number line.

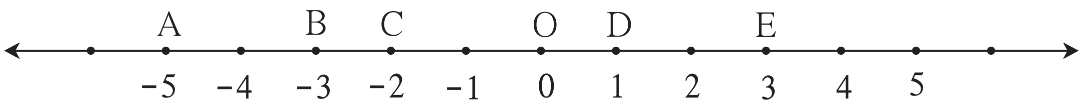


Fig. 1.1

Here, the point D on the number line denotes the number 1. So, it is said that 1 is the co-ordinate of point D. The point B denotes the number – 3 on the line. Hence the co-ordinate of point B is – 3. Similarly the co-ordinates of point A and E are – 5 and 3 respectively.

The point E is 2 unit away from point D. It means the distance between points D and E is 2. Thus, we can find the distance between two points on a number line by counting number of units. The distance between points A and B on the above number line is also 2.

Now let us see how to find distance with the help of co-ordinates of points.

To find the distance between two points, consider their co-ordinates and subtract the smaller co-ordinate from the larger.

The co-ordinates of points D and E are 1 and 3 respectively. We know that $3 > 1$.

Therefore, distance between points E and D $= 3 - 1 = 2$

The distance between points E and D is denoted as d (E,D). This is the same as l(ED), that is, the length of the segment ED.

d (E, D) $= 3 - 1 = 2$

$\therefore$ l(ED) $= 2$

d (E, D) $= l$(ED) $= 2$

Similarly d (D, E) $= 2$

d (C, D) $= 1 - (-2)$

$= 1 + 2 = 3$

$\therefore$ d (C, D) $= l$(CD) $= 3$

Similarly d (D, C) $= 3$

Now, let us find d(A,B). The co-ordinate of A is – 5 and that of B is – 3; $-3 > -5$

$\therefore$ d (A, B) $= -3 - (-5) = -3+5 = 2$.

From the above examples it is clear that the distance between two distinct points is always a positive number.

Note that, if the two points are not distinct then the distance between them is zero.

Remember this !

- The distance between two points is obtained by subtracting the smaller co-ordiante from the larger co-ordinate.
- The distance between any two points is a non-negative real number.

Let's learn.

Betweenness

If P, Q, R are three distinct collinear points, there are three possibilities.

Fig. 1.2

(i) Point Q is between P and R

(ii) Point R is between P and Q

(iii) Point P is between R and Q

If d (P, Q) + d (Q, R) = d (P, R) then it is said that point Q is between P and R. The betweeness is shown as P - Q - R.

Solved examples

Ex (1) On a number line, points A, B and C are such that

d (A, B) = 5, d (B,C) = 11 and d (A, C) = 6.

Which of the points is between the other two ?

Solution : Which of the points A, B and C is between the other two, can be decided as follows.

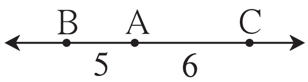


Fig. 1.3

d(B,C) = 11 . . . . (I)

d(A,B) + d(A,C)= 5+6 = 11 . . . . (II)

$\therefore$ d (B, C) = d (A, B) + d (A, C) . . . . [from (I) and (II)]

Point A is between point B and point C.

Ex (2) U, V and A are three cities on a straight road. The distance between U and A is 215 km, between V and A is 140 km and between U and V is 75 km. Which of them is between the other two ?

Solution : d (U,A) = 215; d (V,A) = 140; d (U,V) = 75

d (U,V) + d (V,A) = 75 + 140 = 215; d (U,A) = 215

$\therefore$ d (U,A) = d (U,V) + d (V,A)

$\therefore$ The city V is between the cities U and A.

Ex (3) The co-ordinate of point A on a number line is 5. Find the co-ordinates of points on the same number line which are 13 units away from A.

Solution : As shown in the figure, let us take points T and D to the left and right of A respectively, at a distance of 13 units.

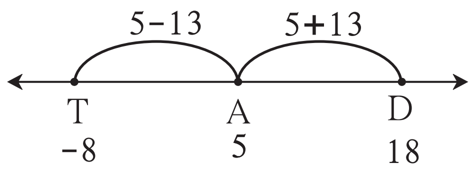


Fig. 1.4

The co-ordinate of point T, which is to the left of A ,will be 5 – 13 = – 8

The co-ordinate of point D, which is to the right of A, will be 5 + 13 = 18

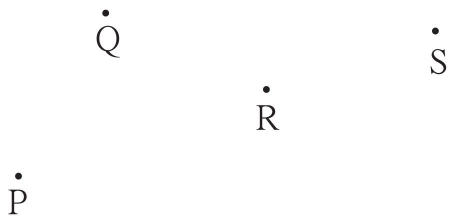
∴ the co-ordinates of points 13 units away from A will be – 8 and 18.

Verify your answer : d (A,D) = d(A,T) = 13

Activity

(1) Points A, B, C are given aside. Check, with a stretched thread, whether the three points are collinear or not. If they are collinear, write which one of them is between the other two.

(2) Given aside are four points P, Q, R, and S. Check which three of them are collinear and which three are non collinear. In the case of three collinear points, state which of them is between the other two.



(3) Students are asked to stand in a line for mass drill. How will you check whether the students standing are in a line or not ?

(4) How had you verified that light rays travel in a straight line ?

Recall an experiment in science which you have done in a previous standard.

Practice set 1.1

1. Find the distances with the help of the number line given below.

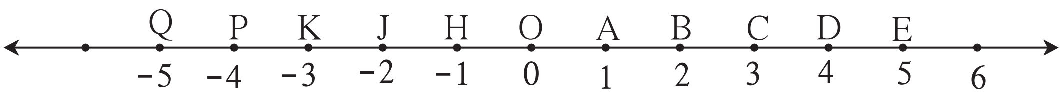


Fig. 1.5

(i) *d*(B,E) (ii) *d*(J, A) (iii) *d*(P, C) (iv) *d*(J, H)

(v) *d*(K, O) (vi) *d*(O, E) (vii) *d*(P, J) (viii) *d*(Q, B)

2. If the co-ordinate of A is x and that of B is y, find *d*(A, B) .

(i) $x = 1, y = 7$ (ii) $x = 6, y = -2$ (iii) $x = -3, y = 7$

(iv) $x = -4, y = -5$ (v) $x = -3, y = -6$ (vi) $x = 4, y = -8$

3. From the information given below, find which of the point is between the other two. If the points are not collinear, state so.

(i) *d*(P, R) = 7, *d*(P, Q) = 10, *d*(Q, R) = 3

(ii) *d*(R, S) = 8, *d*(S, T) = 6, *d*(R, T) = 4

(iii) *d*(A, B) = 16, *d*(C, A) = 9, *d*(B, C) = 7

(iv) *d*(L, M) = 11, *d*(M, N) = 12, *d*(N, L) = 8

(v) *d*(X, Y) = 15, *d*(Y, Z) = 7, *d*(X, Z) = 8

(vi) *d*(D, E) = 5, *d*(E, F) = 8, *d*(D, F) = 6

4. On a number line, points A, B and C are such that *d*(A,C) = 10, *d*(C,B) = 8 Find *d*(A, B) considering all possibilities.

5. Points X, Y, Z are collinear such that *d*(X,Y) = 17, *d*(Y,Z) = 8, find *d*(X,Z) .

6. Sketch proper figure and write the answers of the following questions.

(i) If A - B - C and *l*(AC) = 11, *l*(BC) = 6.5, then *l*(AB) =?

(ii) If R - S - T and *l*(ST) = 3.7, *l*(RS) = 2.5, then *l*(RT) =?

(iii) If X - Y - Z and *l*(XZ) = $3\sqrt{7}$, *l*(XY) = $\sqrt{7}$, then *l*(YZ) =?

7. Which figure is formed by three non-collinear points ?

Let's learn.

In the book, Mathematics - Part I for std IX, we have learnt union and intersection of sets in the topic on sets. Now, let us describe a segment, a ray and a line as sets of points.

(1) Line segment :

The union set of point A, point B and points between A and B is called segment AB. Segment AB is written as seg AB in brief. Seg AB means seg BA. Point A and point B are called the end points of seg AB. The distance between the end points of a segment is called the length of the segment. That is l(AB) = d (A,B)

l(AB) = 5 is also written as AB = 5.

Fig. 1.6

(2) Ray AB :

Suppose, A and B are two distinct points. The union set of all points on seg AB and the points P such that A - B - P, is called ray AB. Here point A is called the starting point of ray AB.

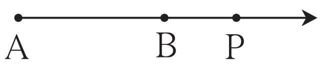


Fig. 1.7

(3) Line AB :

The union set of points on ray AB and opposite ray of ray AB is called line AB. The set of points of seg AB is a subset of points of line AB.

(4) Congruent segments :

If the length of two segments is equal then the two segments are congruent.

If l(AB) = l(CD) then seg AB ≅ seg CD

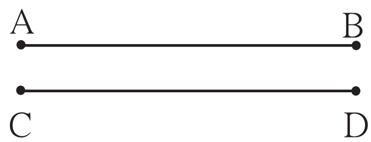


Fig. 1.8

(5) Properties of congruent segements :

(i) Reflexivity : seg AB ≅ seg AB

(ii) Symmetry : If seg AB ≅ seg CD then seg CD ≅ seg AB

(iii) Transitivity : If seg AB ≅ seg CD and seg CD ≅ seg EF then seg AB ≅ seg EF

(6) Midpoint of a segment :

If A-M-B and seg AM ≅ seg MB, then M is called the midpoint of seg AB.

Every segment has one and only one midpoint.

Fig. 1.9

(7) **Comparison of segments :**

If length of segment AB is less than the length of segment CD, it is written as seg AB < seg CD or seg CD > seg AB.

The comparison of segments depends upon their lengths.

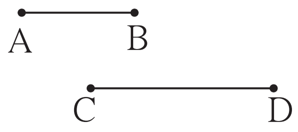


Fig. 1.10

(8) **Perpendicularity of segments or rays :**

If the lines containing two segments, two rays or a ray and a segment are perpendicular to each other then the two segments, two rays or the segment and the ray are said to be perpendicular to each other.

In the figure 1.11, seg AB $\perp$ line CD,

seg AB $\perp$ ray CD.

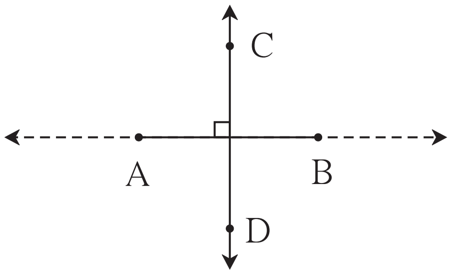


Fig. 1.11

(9) **Distance of a point from a line :**

If seg CD $\perp$ line AB and the point D lies on line AB then the length of seg CD is called the distance of point C from line AB.

The point D is called the foot of the perpendicular.

If l(CD) = a, then the point C is at a distance of 'a' from the line AB.

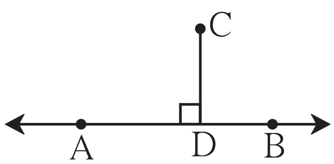


Fig. 1.12

Practice set 1.2

1. The following table shows points on a number line and their co-ordinates. Decide whether the pair of segments given below the table are congruent or not.

Point	A	B	C	D	E
Co-ordinate	–3	5	2	–7	9

(i) seg DE and seg AB (ii) seg BC and seg AD (iii) seg BE and seg AD

2. Point M is the midpoint of seg AB. If AB = 8 then find the length of AM.

3. Point P is the midpoint of seg CD. If CP = 2.5, find l(CD).

4. If AB = 5 cm, BP = 2 cm and AP = 3.4 cm, compare the segments.

5. Write the answers to the following questions with reference to figure 1.13.

(i) Write the name of the opposite ray of ray RP

(ii) Write the intersection set of ray PQ and ray RP.

(iii) Write the union set of seg PQ and seg QR.

(iv) State the rays of which seg QR is a subset.

(v) Write the pair of opposite rays with common end point R.

(vi) Write any two rays with common end point S.

(vii) Write the intersection set of ray SP and ray ST.

Fig. 1.13

6. Answer the questions with the help of figure 1.14.

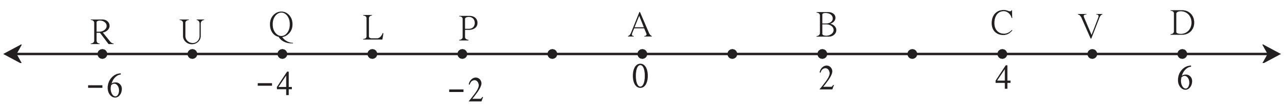


Fig. 1.14

(i) State the points which are equidistant from point B.

(ii) Write a pair of points equidistant from point Q.

(iii) Find d (U,V), d (P,C), d (V,B), d (U, L).

Let's learn.

Conditional statements and converse

The statements which can be written in the 'If-then' form are called conditional statements. The part of the statement following 'If' is called the antecedent, and the part following 'then' is called the consequent.

For example, consider the statement : The diagonals of a rhombus are perpendicular bisectors of each other.

The statement can be written in the conditional form as, 'If the given quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.'

If the antecedent and consequent in a given conditional statement are interchanged, the resulting statement is called the **converse** of the given statement.

If a conditional statement is true, its converse is not necessarily true. Study the following examples.

Conditional statement : If a quadrilateral is a rhombus then its diagonals are perpendicular bisectors of each other.

Converse : If the diagonals of a quadrilateral are perpendicular bisectors of each other then it is a rhombus.

In the above example, the statement and its converse are true.

Now consider the following example,

Conditional statement : If a number is a prime number then it is even or odd.

Converse : If a number is even or odd then it is a prime number.

In this example, the statement is true, but its converse is false.

Let's learn.

Proofs

We have studied many properties of angles, triangles and quadrilaterals through activities.

In this standard we are going to look at the subject of Geometry with a different point of view, which was originated by the Greek mathematician Euclid, who lived in the third century before Christian Era. He gathered the knowledge of Geometry prevailing at his time and streamlined it. He took for granted some self evident geometrical statements which were accepted by all and called them **Postulates.** He showed that on the basis of the postulates some more properties can be proved logically.

Properties proved logically are called **Theorems.**

Some of Euclid's postulates are given below.

(1) There are infinite lines passing through a point.

(2) There is one and only one line passing through two points.

(3) A circle of given radius can be drawn taking any point as its centre.

(4) All right angles are congruent with each other.

(5) If two interior angles formed on one side of a transversal of two lines add up to less than two right angles then the lines produced in that direction intersect each other.

Euclid

We have verified some of these postulates through activities.

A property is supposed to be true if it can be proved logically. It is then called a **Theorem.** The logical argument made to prove a theorem is called its **proof.**

When we are going to prove that a conditional statement is true, its antecedent is called 'Given part' and the consequent is called 'the part to be proved'.

There are two types of proofs, **Direct** and **Indirect.**

Let us give a direct proof of the property of angles made by two intersecting lines.

Theorem : The opposite angles formed by two intersecting lines are of equal measures.

Given : Line AB and line CD intersect at point O such that A - O - B, C - O - D.

To prove : (i) $\angle AOC = \angle BOD$

(ii) $\angle BOC = \angle AOD$

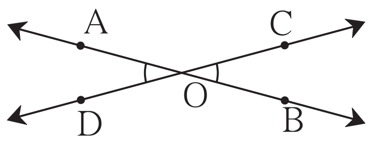


Fig. 1.15

Proof : $\angle AOC + \angle BOC = 180^\circ$ . . . . (I) (angles in linear pair)

$\angle BOC + \angle BOD = 180^\circ$ . . . . (II) (angles in linear pair)

$\angle AOC + \angle BOC = \angle BOC + \angle BOD$ . . . . [from (I) and (II)]

$\therefore$ $\angle AOC = \angle BOD$. . . . . . . eliminating $\angle BOC$.

Similarly, it can be proved that $\angle BOC = \angle AOD$.

Indirect proof :

This type of proof starts with an assumption that the consequence is false. Using it and the properties accepted earlier, we start arguing step by step and reach a conclusion. The conclusion is contradictory with the antecedent or a property which is already accepted. Hence, the assumption that the consequent is false goes wrong. So it is accepted that the consequent is true.

Study the following example.

Statement : A prime number greater than 2 is odd.

Conditional statement : If *p* is a prime number greater than 2 then it is odd.

Given : *p* is a prime number greater than 2. That is, 1 and *p* are the only divisors of *p*.

To prove : *p* is an odd number.

Proof : Let us suppose that *p* is not an odd number.

So *p* is an even number

$\therefore$ a divisor of *p* is 2 ..... (I)

But it is given that *p* is a prime number greater than 2 ....(given)

$\therefore$ 1 and *p* are the only divisors of *p* ..... (II)

Statements (I) and (II) are contradictory.

$\therefore$ the assumption , that *p* is not odd is false.

This proves that a prime number greater than 2 is odd.

Practice set 1.3

1. Write the following statements in 'if-then' form.

(i) The opposite angles of a parallelogram are congruent.

(ii) The diagonals of a rectangle are congruent.

(iii) In an isosceles triangle, the segment joining the vertex and the mid point of the base is perpendicular to the base.

2. Write converses of the following statements.

(i) The alternate angles formed by two parallel lines and their transversal are congruent.

(ii) If a pair of the interior angles made by a transversal of two lines are supplementary then the lines are parallel.

(iii) The diagonals of a rectangle are congruent.

Problem set 1

1. Select the correct alternative from the answers of the questions given below.

(i) How many mid points does a segment have ?

(A) only one (B) two (C) three (D) many

(ii) How many points are there in the intersection of two distinct lines ?

(A) infinite (B) two (C) one (D) not a single

(iii) How many lines are determined by three distinct points ?

(A) two (B) three (C) one or three (D) six

(iv) Find d(A, B), if co-ordinates of A and B are – 2 and 5 respectively.

(A) – 2 (B) 5 (C) 7 (D) 3

(v) If P - Q - R and d(P,Q) = 2, d(P,R) = 10, then find d(Q,R).

(A) 12 (B) 8 (C) $\sqrt{96}$ (D) 20

2. On a number line, co-ordinates of P, Q, R are 3, – 5 and 6 respectively. State with reason whether the following statements are true or false.

(i) d(P,Q) + d(Q,R) = d(P,R)

(ii) d(P,R) + d(R,Q) = d(P,Q)

(iii) d(R,P) + d(P,Q) = d(R,Q)

(iv) d(P,Q) – d(P,R) = d(Q,R)

3. Co-ordinates of some pairs of points are given below. Hence find the distance between each pair.

(i) 3, 6 (ii) – 9, – 1 (iii) – 4, 5 (iv) 0, – 2

(v) x + 3, x – 3 (vi) – 25, – 47 (vii) 80, – 85

4. Co-ordinate of point P on a number line is – 7. Find the co-ordinates of points on the number line which are at a distance of 8 units from point P.

5. Answer the following questions.

(i) If A - B - C and d(A,C) = 17, d(B,C) = 6.5 then d (A,B) = ?

(ii) If P - Q - R and d(P,Q) = 3.4, d(Q,R)= 5.7 then d(P,R) = ?

6. Co-ordinate of point A on a number line is 1. What are the co-ordinates of points on the number line which are at a distance of 7 units from A ?

7. Write the following statements in conditional form.

(i) Every rhombus is a square.

(ii) Angles in a linear pair are supplementary.

(iii) A triangle is a figure formed by three segments.

(iv) A number having only two divisors is called a prime number.

8. Write the converse of each of the following statements.

(i) If the sum of measures of angles in a figure is 180^0, then the figure is a triangle.

(ii) If the sum of measures of two angles is 90^0 then they are complement of each other.

(iii) If the corresponding angles formed by a transversal of two lines are congruent then the two lines are parallel.

(iv) If the sum of the digits of a number is divisible by 3 then the number is divisible by 3.

9. Write the antecedent (given part) and the consequent (part to be proved) in the following statements.

(i) If all sides of a triangle are congruent then its all angles are congruent.

(ii) The diagonals of a parallelogram bisect each other.

10*. Draw a labelled figure showing information in each of the following statements and write the antecedent and the consequent.

(i) Two equilateral triangles are similar.

(ii) If angles in a linear pair are congruent then each of them is a right angle.

(iii) If the altitudes drawn on two sides of a triangle are congruent then those two sides are congruent.

❑❑❑

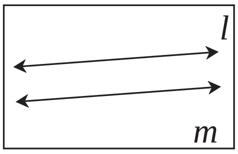
2 Parallel Lines

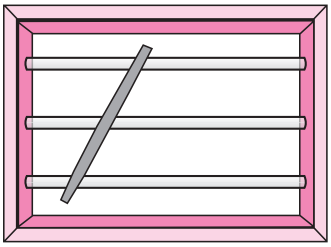
Let's study.

- Properties of angles formed by parallel lines and its transversal
- Tests of parallelness of two lines
- Use of properties of parallel lines

Let's recall.

Parallel lines : The lines which are coplanar and do not intersect each other are called parallel lines.





Hold a stick across the horizontal parallel bars of a window as shown in the figure. How many angles are formed ?

- Do you recall the pairs of angles formed by two lines and their transversal ?

  In figure 2.1, line n is a transversal of line l and line m.

  Here, in all 8 angles are formed. Pairs of angles formed out of these angles are as follows :

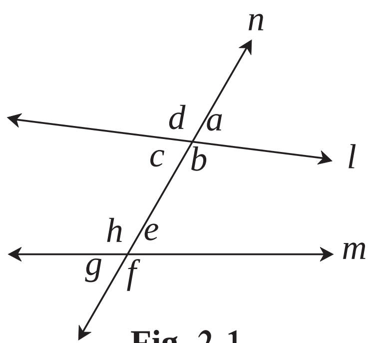


Fig. 2.1

Pairs of corresponding angles

(i) $\angle d, \angle h$

(ii) $\angle a$, □

(iii) $\angle c$, □

(iv) $\angle b$, □

Pairs of alternate interior angles

(i) $\angle c, \angle e$ (ii) $\angle b, \angle h$

Pairs of alternate exerior angles

(i) $\angle d, \angle f$

(ii) $\angle a, \angle g$

Pairs of interior angles on the same side of the transversal

(i) $\angle c, \angle h$

(ii) $\angle b, \angle e$

Some important properties :

(1) When two lines intersect, the pairs of opposite angles formed are congruent.

(2) The angles in a linear pair are supplementary.

(3) When one pair of corresponding angles is congruent, then all the remaining pairs of corresponding angles are congruent.

(4) When one pair of alternate angles is congruent, then all the remaining pairs of alternate angles are congruent.

(5) When one pair of interior angles on one side of the transversal is supplementary, then the other pair of interior angles is also supplementary.

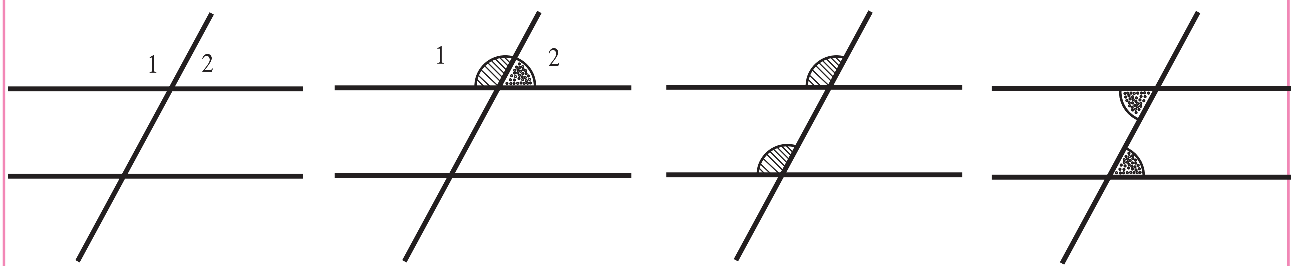
Let's learn.

Properties of parallel lines

Activity

To verify the properties of angles formed by a transversal of two parallel lines.

Take a piece of thick coloured paper. Draw a pair of parallel lines and a transversal on it. Paste straight sticks on the lines. Eight angles will be formed. Cut pieces of coloured paper, as shown in the figure, which will just fit at the corners of ∠1 and ∠2. Place the pieces near different pairs of corresponding angles, alternate angles and interior angles and verify their properties.



Let's learn.

We have verified the properties of angles formed by a transversal of two parallel lines. Let us now prove the properties using Euclid's famous fifth postulate given below.

If sum of two interior angles formed on one side of a transversal of two lines is less than two right angles then the lines produced in that direction intersect each other.

Interior angle theorem

Theorem : If two parallel lines are intersected by a transversal, the interior angles on either side of the transversal are supplementary.

Given : line $l \parallel$ line m and line n is their transversal. Hence as shown in the figure $\angle a$, $\angle b$ are interior angles formed on one side and $\angle c$, $\angle d$ are interior angles formed on other side of the transversal.

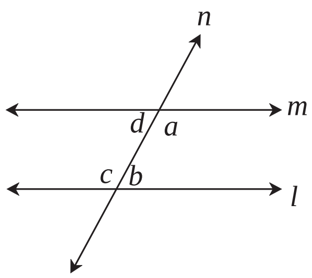


Fig. 2.2

To prove : $\angle a + \angle b = 180°$

$\angle d + \angle c = 180°$

Proof : Three possibilities arise regarding the sum of measures of $\angle a$ and $\angle b$.

(i) $\angle a + \angle b < 180°$ (ii) $\angle a + \angle b > 180°$ (iii) $\angle a + \angle b = 180°$

Let us assume that the possibility (i) $\angle a + \angle b < 180°$ is true.

Then according to Euclid's postulate, if the line l and line m are produced will intersect each other on the side of the transversal where $\angle a$ and $\angle b$ are formed.

But line l and line m are parallel lines ....given

$\therefore \angle a + \angle b < 180°$ impossible ....(I)

Now let us suppose that $\angle a + \angle b > 180°$ is true.

$\therefore \angle a + \angle b > 180°$

But $\angle a + \angle d = 180°$

and $\angle c + \angle b = 180°$ . . . . . angles in linear pairs

$\therefore \angle a + \angle d + \angle b + \angle c = 180° + 180° = 360°$

$\therefore \angle c + \angle d = 360° - (\angle a + \angle b)$

If $\angle a + \angle b > 180°$ then $[360° - (\angle a + \angle b)] < 180°$

$\therefore \angle c + \angle d < 180°$

$\therefore$ In that case line l and line m produced will intersect each other on the same side of the transversal where $\angle c$ and $\angle d$ are formed.

$\therefore \angle c + \angle d < 180$ is impossible.

That is $\angle a + \angle b > 180^\circ$ is impossible...... (II)

$\therefore$ the remaining possibility,

$\angle a + \angle b = 180^\circ$ is true......from (I) and (II)

$\therefore$ $\angle a + \angle b = 180^\circ$ Similarly, $\angle c + \angle d = 180^\circ$

Note that, in this proof, because of the contradictions we have denied the possibilities $\angle a + \angle b > 180^\circ$ and $\angle a + \angle b < 180^\circ$.

Therefore, this proof is an example of indirect proof.

Corresponding angles and alternate angles theorems

Theorem : The corresponding angles formed by a transversal of two parallel lines are of equal measure.

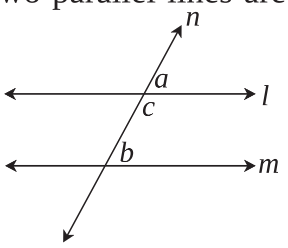


Fig. 2.3

Given : line l || line m
line n is a transversal.

To prove : $\angle a = \angle b$

Proof : $\angle a + \angle c = 180^\circ$ ....(I) angles in linear pair

$\angle b + \angle c = 180^\circ$ ....(II) property of interior angles of parallel lines

$\angle a + \angle c = \angle b + \angle c$ ....from (I) and (II)

$\therefore$ $\angle a = \angle b$

Theorem : The alternate angles formed by a transversal of two parallel lines are of equal measures.

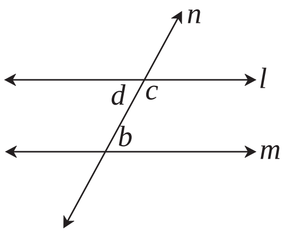


Fig. 2.4

Given : line l || line m
line n is a transversal.

To prove : $\angle d = \angle b$

Proof : $\angle d + \angle c = 180^\circ$................(I) angles in linear pair

$\angle c + \angle b = 180^\circ$ ....(II) property of interior angles of parallel line

$\angle d + \angle c = \angle c + \angle b$................from (I) and (II)

$\therefore$ $\angle d = \angle b$

Practice set 2.1

1. In figure 2.5, line RP || line MS and line DK is their transversal. $\angle DHP = 85^\circ$
Find the measures of following angles.
(i) $\angle RHD$ (ii) $\angle PHG$
(iii) $\angle HGS$ (iv) $\angle MGK$

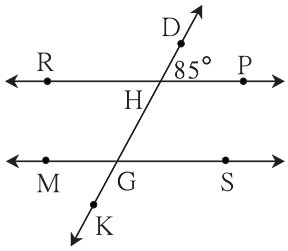


Fig. 2.5

2. In figure 2.6, line *p* || line *q* and line *l* and line *m* are transversals. Measures of some angles are shown. Hence find the measures of
$\angle a$, $\angle b$, $\angle c$, $\angle d$.

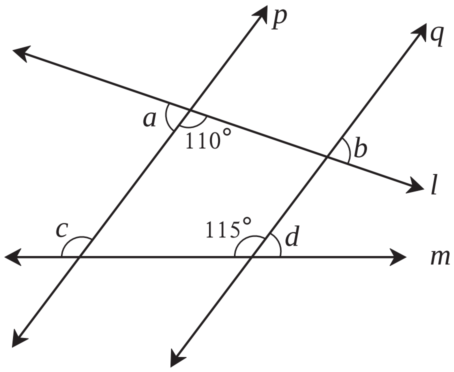


Fig. 2.6

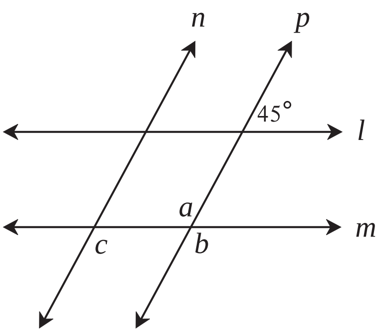


Fig. 2.7

3. In figure 2.7, line *l* || line *m* and line *n* || line *p*. Find $\angle a$, $\angle b$, $\angle c$ from the given measure of an angle.

4*. In figure 2.8, sides of $\angle PQR$ and $\angle XYZ$ are parallel to each other. Prove that, $\angle PQR \cong \angle XYZ$

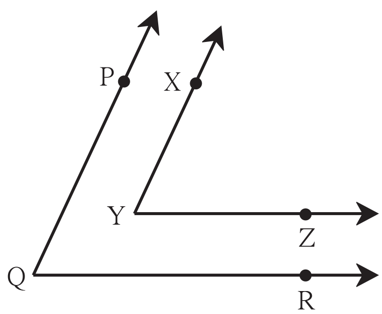


Fig. 2.8

5. In figure 2.9, line AB || line CD and line PQ is transversal. Measure of one of the angles is given. Hence find the measures of the following angles.

(i) ∠ART (ii) ∠CTQ

(iii) ∠DTQ (iv) ∠PRB

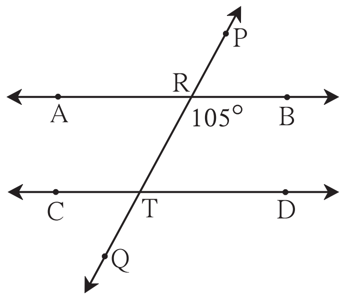


Fig. 2.9

Let's learn.

Use of properties of parallel lines

Let us prove a property of a triangle using the properties of angles made by a transversal of parallel lines.

Theorem : The sum of measures of all angles of a triangle is 180°.

Given : Δ ABC is any triangle.

To prove : ∠ABC + ∠ACB + ∠BAC = 180°.

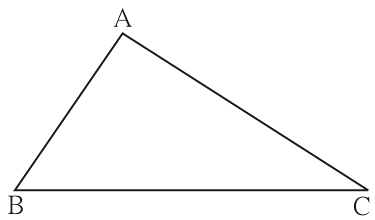


Fig. 2.10

Construction : Draw a line parallel to seg BC and passing through A. On the line take points P and Q such that, P - A - Q.

Proof : Line PQ || line BC and seg AB is a transversal.

∴ ∠ABC = ∠PAB.......alternate angles.....(I)

line PQ || line BC and seg AC is a transversal.

∴ ∠ACB = ∠QAC.......alternate angles.....(II)

∴ From I and II ,

∠ABC + ∠ACB = ∠PAB + ∠QAC . . . (III)

Adding ∠BAC to both sides of (III).

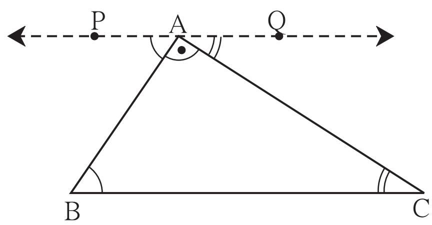


Fig. 2.11

∠ABC + ∠ACB + ∠BAC = ∠PAB + ∠QAC + ∠BAC

= ∠PAB + ∠BAC + ∠QAC

= ∠PAC + ∠QAC ...(∵ ∠PAB + ∠BAC = ∠PAC)

= 180° ...Angles in linear pair

That is, sum of measures of all three angles of a triangle is 180°.

Let's discuss.

In fig. 2.12, How will you decide whether line *l* and line *m* are parallel or not ?

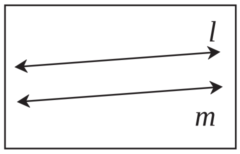


Fig. 2.12

Let's learn.

Tests for parallel lines

Whether given two lines are parallel or not can be decided by examining the angles formed by a transversal of the lines.

(1) If the interior angles on the same side of a transversal are supplementary then the lines are parallel.

(2) If one of the pairs of alternate angles is congruent then the lines are parallel.

(3) If one of the pairs of corresponding angles is congruent then the lines are parallel.

Interior angles test

Theorem : If the interior angles formed by a transversal of two distinct lines are supplementary, then the two lines are parallel.

Given : Line XY is a transversal of line AB and line CD.

$\angle BPQ + \angle PQD = 180^\circ$

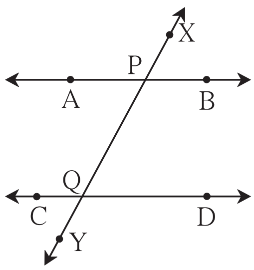


Fig. 2.13

To prove : line AB $\|$ line CD

Proof : We are going to give an indirect proof.

Let us suppose that the statement to be proved is wrong. That is, we assume,

line AB and line CD are not parallel,

means line AB and CD intersect at point T.

So Δ PQT is formed.

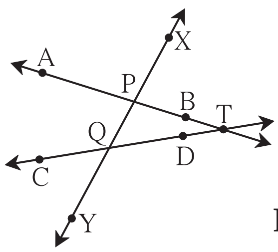


Fig. 2.14

$\therefore$ $\angle TPQ + \angle PQT + \angle PTQ = 180^\circ$ ....sum of angles of a triangle

but $\angle TPQ + \angle PQT = 180^\circ$ ....given

That is the sum of two angles of the triangle is 180°.

But sum of three angles of a triangle is 180°.

$\therefore$ $\angle PTQ = 0^\circ$.

$\therefore$ line PT and line QT means line AB and line CD are not distinct lines.

But, we are given that line AB and line CD are distinct lines.

$\therefore$ we arrive at a contradiction.

$\therefore$ our assumption is wrong. Hence line AB and line CD are parallel.

Thus it is proved that if the interior angles formed by a transversal are supplementary, then the lines are parallel.

This property is called **interior angles test** of parallel lines.

Alternate angles test

Theorem : If a pair of alternate angles formed by a transversal of two lines is congruent then the two lines are parallel.

Given : Line n is a transversal of line l and line m.

$\angle a$ and $\angle b$ is a congruent pair of alternate angles.

That is, $\angle a = \angle b$

To prove : line $l \parallel$ line m

Proof : $\angle a + \angle c = 180^\circ$ . ....angles in linear pair

$\angle a = \angle b$ .... given

$\therefore \angle b + \angle c = 180^\circ$

But $\angle b$ and $\angle c$ are interior angles on the same side of the transversal.

$\therefore$ line $l \parallel$ line m .... interior angles test

This property is called the **alternate angles test** of parallel lines.

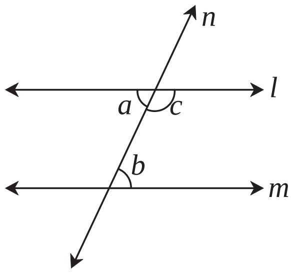


Fig. 2.15

Corresponding angles Test

Theorem : If a pair of corresponding angles formed by a transversal of two lines is congruent then the two lines are parallel.

Given : Line n is a transversal of line l and line m.

$\angle a$ and $\angle b$ is a congruent pair of corresponding angles.

That is, $\angle a = \angle b$

To prove : line $l \parallel$ line m

Proof : $\angle a + \angle c = 180^\circ$ ....angles in linear pair

$\angle a = \angle b$ .... given

$\therefore \angle b + \angle c = 180^\circ$

That is a pair of interior angles on the same side of the transversal is congruent.

$\therefore$ line $l \parallel$ line m ....interior angles test

This property is called the **corresponding angles test** of parallel lines.

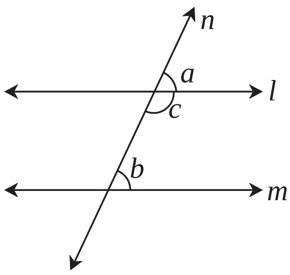


Fig. 2.16

Corollary I : If a line is perpendicular to two lines in a plane, then the two lines are parallel to each other.

Given : Line $n \perp$ line l and line $n \perp$ line m

To prove : line $l \parallel$ line m

Proof : line $n \perp$ line l and line $n \perp$ line m ...given

$\therefore \angle a = \angle c = 90°$

$\angle a$ and $\angle c$ are corresponding angles formed by transversal n of line l and line m.

$\therefore$ line $l \parallel$ line m ....corresponding angles test

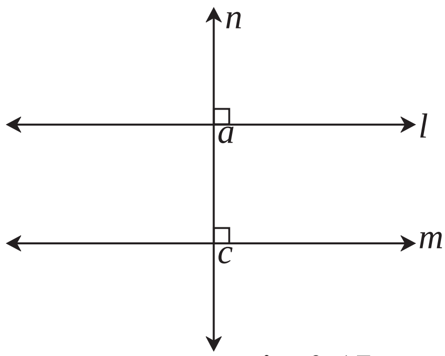


Fig. 2.17

Corollary II : If two lines in a plane are parallel to a third line in the plane then those two lines are parallel to each other. Write the proof of the corollary.

Practice set 2.2

1. In figure 2.18, $y = 108°$ and $x = 71°$
Are the lines m and n parallel ? Justify ?

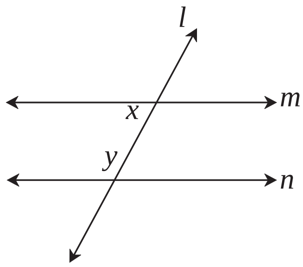


Fig. 2.18

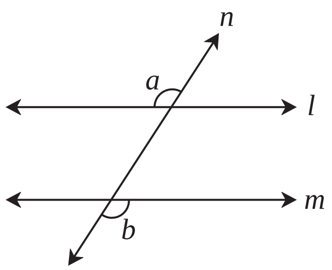


Fig. 2.19

2. In figure 2.19, if $\angle a \cong \angle b$ then prove that line $l \parallel$ line m.

3. In figure 2.20, if $\angle a \cong \angle b$ and $\angle x \cong \angle y$ then prove that line $l \parallel$ line n.

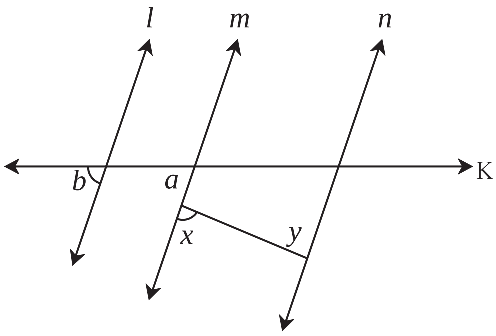


Fig. 2.20

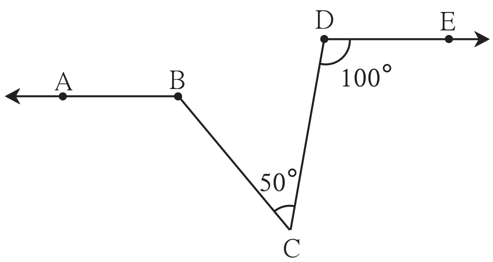


Fig. 2.21

4. In figure 2.21, if ray BA $\parallel$ ray DE, $\angle C = 50°$ and $\angle D = 100°$. Find the measure of $\angle ABC$.
(Hint : Draw a line passing through point C and parallel to line AB.)

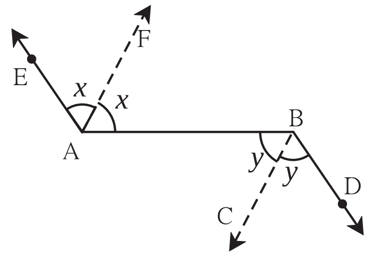
5. 

Fig. 2.22

In figure 2.22, ray AE || ray BD, ray AF is the bisector of $\angle$EAB and ray BC is the bisector of $\angle$ABD. Prove that line AF || line BC.

6. A transversal EF of line AB and line CD intersects the lines at point P and Q respectively. Ray PR and ray QS are parallel and bisectors of $\angle$BPQ and $\angle$PQC respectively.
Prove that line AB || line CD.

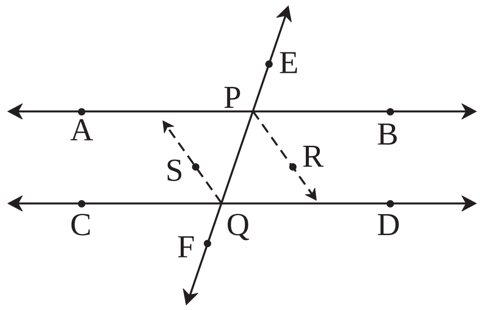


Fig. 2.23

Problem set 2

1. Select the correct alternative and fill in the blanks in the following statements.
   (i) If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is ....
   (A) 0° (B) 90° (C) 180° (D) 360°
   (ii) The number of angles formed by a transversal of two lines is ....
   (A) 2 (B) 4 (C) 8 (D) 16
   (iii) A transversal intersects two parallel lines. If the measure of one of the angles is 40° then the measure of its corresponding angle is ....
   (A) 40° (B) 140° (C) 50° (D) 180°
   (iv) In Δ ABC, $\angle A = 76^\circ$, $\angle B = 48^\circ$, $\therefore \angle C =$ ....
   (A) 66° (B) 56° (C) 124° (D) 28°
   (v) Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is 75° then the measure of the other angle is ....
   (A) 105° (B) 15° (C) 75° (D) 45°

2*. Ray PQ and ray PR are perpendicular to each other. Points B and A are in the interior and exterior of $\angle$QPR respectively. Ray PB and ray PA are perpendicular to each other. Draw a figure showing all these rays and write -
(i) A pair of complementary angles (ii) A pair of supplementary angles.
(iii) A pair of congruent angles.

3. Prove that, if a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other line also.

4. In figure 2.24, measures of some angles are shown. Using the measures find the measures of $\angle x$ and $\angle y$ and hence show that line $l \parallel$ line m.

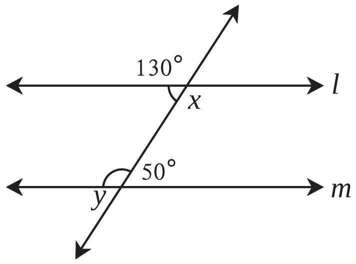


Fig. 2.24

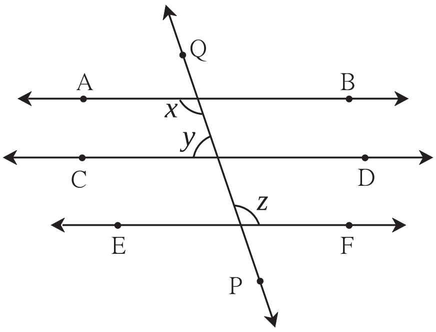


Fig. 2.25

5. Line AB $\parallel$ line CD $\parallel$ line EF and line QP is their transversal. If $y : z = 3 : 7$ then find the measure of $\angle x$. (See figure 2.25.)

6. In figure 2.26, if line $q \parallel$ line r, line p is their transversal and if $a = 80°$ find the values of f and g.

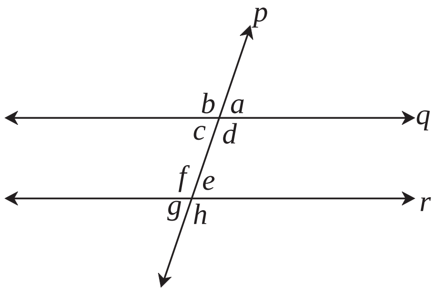


Fig. 2.26

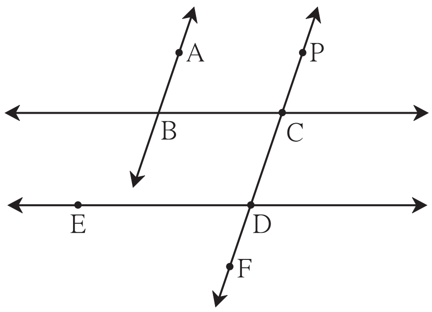


Fig. 2.27

7. In figure 2.27, if line AB $\parallel$ line CF and line BC $\parallel$ line ED then prove that $\angle$ABC = $\angle$FDE.

8. In figure 2.28, line PS is a transversal of parallel line AB and line CD. If Ray QX, ray QY, ray RX, ray RY are angle bisectors, then prove that □ QXRY is a rectangle.

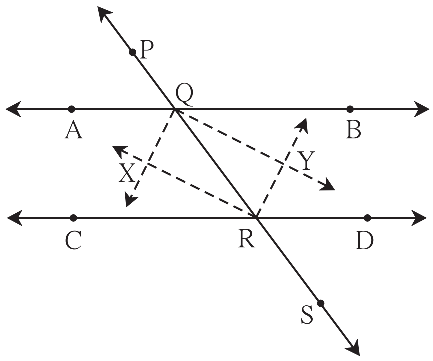


Fig. 2.28

❑❑❑

3 Triangles

23NCHF

Let's study.

- Theorem of remote interior angles of a triangle
- Congruence of triangles
- Theorem of an isosceles triangle
- Property of 30°- 60°- 90° angled triangle
- Median of a triangle
- Property of median on hypotenuse of a right angled triangle
- Perpendicular bisector theorem
- Angle bisector theorem
- Similar triangles

Activity :

Draw a triangle of any measure on a thick paper. Take a point T on ray QR as shown in fig. 3.1. Cut two pieces of thick paper which will exactly fit the corners of ∠P and ∠Q. See that the same two pieces fit exactly at the corner of ∠PRT as shown in the figure.

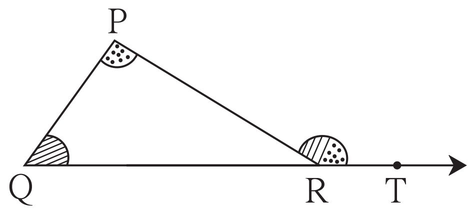


Fig. 3.1

Let's learn.

Theorem of remote interior angles of a triangle

Theorem : The measure of an exterior angle of a triangle is equal to the sum of its remote interior angles.

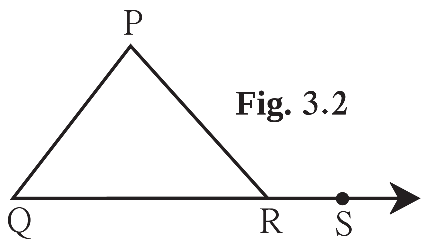


Fig. 3.2

Given : ∠PRS is an exterior angle of Δ PQR.

To prove : ∠PRS = ∠PQR + ∠QPR

Proof : The sum of all angles of a triangle is 180°.

∴ ∠PQR + ∠QPR + ∠PRQ = 180°.......(I)

∠PRQ + ∠PRS = 180°.....angles in linear pair......(II)

∴ from (I) and (II)

∠PQR + ∠QPR + ∠PRQ = ∠PRQ + ∠PRS

∴ ∠PQR + ∠QPR = ∠PRS ....eliminating ∠PRQ from both sides

∴ the measure of an exterior angle of a triangle is equal to the sum of its remote interior angles.

Use your brain power! Can we give an alternative proof of the theorem drawing a line through point R and parallel to seg PQ in figure 3.2 ?

Let's learn.

Property of an exterior angle of triangle

The sum of two positive numbers a and b, that is $(a + b)$ is greater than a and greater than b also. That is, $a + b > a$, $a + b > b$

Using this inequality we get one property relaed to exterior angle of a triangle.

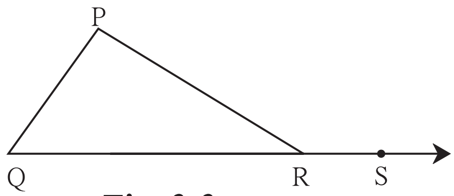


Fig. 3.3

If $\angle PRS$ is an exterior angle of Δ PQR then

$\angle PRS > \angle P$, $\angle PRS > \angle Q$

$\therefore$ an exterior angle of a triangle is greater than its remote interior angle.

Solved examples

Ex (1) The measures of angles of a triangle are in the ratio 5 **:** 6 **:** 7. Find the measures.

Solution : Let the measures of the angles of a triangle be $5x$, $6x$, $7x$.

$$\therefore \quad 5x + 6x + 7x = 180^\circ$$

$$18x = 180^\circ$$

$$x = 10^\circ$$

$5x = 5 \times 10 = 50^\circ$, $6x = 6 \times 10 = 60^\circ$, $7x = 7 \times 10 = 70^\circ$

$\therefore$ the measures of angles of the triangle are 50°, 60° and 70°.

Ex (2) Observe figure 3.4 and find the measures of $\angle PRS$ and $\angle RTS$.

Solution : $\angle PRS$ is an exterior angle of Δ PQR.

So from the theorem of remote interior angles,

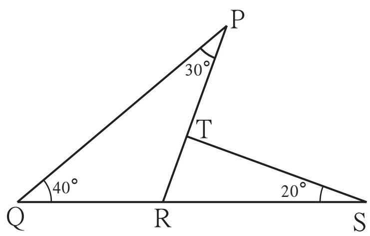


Fig. 3.4

$\angle PRS = \angle PQR + \angle QPR$

$= 40^\circ + 30^\circ$

$= 70^\circ$

In Δ RTS

$\angle TRS + \angle RTS + \angle TSR = \square$ .... sum of all angles of a triangle

$\therefore \square + \angle RTS + \square = 180^\circ$

$\therefore \angle RTS + 90^\circ = 180^\circ$

$\therefore \angle RTS = \square$

Ex (3) Prove that the sum of exterior angles of a triangle, obtained by extending its sides in the same direction is 360°.

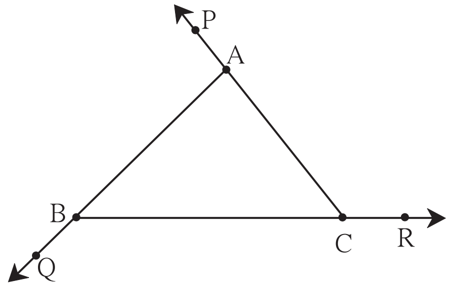


Fig. 3.5

Given : $\angle PAB, \angle QBC$ and $\angle ACR$ are exterior angles of Δ ABC

To prove : $\angle PAB + \angle QBC + \angle ACR = 360^\circ$

Proof : **Method I**

Considering exterior $\angle PAB$ of Δ ABC,

$\angle ABC$ and $\angle ACB$ are its remote interior angles.

$\angle PAB = \angle ABC + \angle ACB$ ----(I)

Similarly, $\angle ACR = \angle ABC + \angle BAC$ ----(II)..theorem of remote interior angles

and $\angle CBQ = \angle BAC + \angle ACB$ ---- (III)

Adding (I), (II) and (III),

$\angle PAB + \angle ACR + \angle CBQ$

$= \angle ABC + \angle ACB + \angle ABC + \angle BAC + \angle BAC + \angle ACB$

$= 2\angle ABC + 2\angle ACB + 2\angle BAC$

$= 2(\angle ABC + \angle ACB + \angle BAC)$

$= 2 \times 180^\circ$ . . . . . sum of interior angles of a triangle

$= 360^\circ$

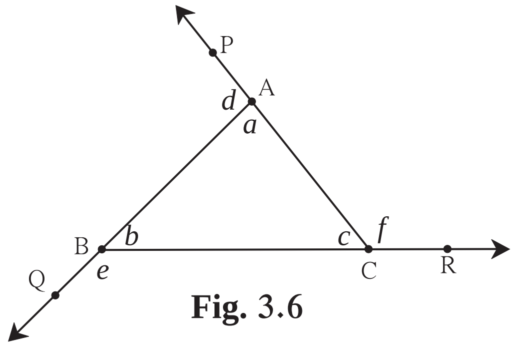


Fig. 3.6

Method II

$\angle c + \angle f = 180^\circ$ . . . . (angles in linear pair)

Also, $\angle a + \angle d = 180^\circ$

and $\angle b + \angle e = 180^\circ$

$\therefore \angle c + \angle f + \angle a + \angle d + \angle b + \angle e = 180^\circ \times 3 = 540^\circ$

$\angle f + \angle d + \angle e + (\angle a + \angle b + \angle c) = 540^\circ$

$\therefore \angle f + \angle d + \angle e + 180^\circ = 540^\circ$

$\therefore f + d + e = 540^\circ - 180^\circ$

$= 360^\circ$

Ex (4) In figure 3.7, bisectors of $\angle B$ and $\angle C$ of Δ ABC intersect at point P.

Prove that $\angle BPC = 90 + \frac{1}{2} \angle BAC$.

Complete the proof filling in the blanks.

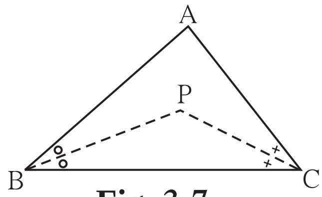


Fig. 3.7

Proof : In Δ ABC,

$\angle BAC + \angle ABC + \angle ACB = \square$ ...... sum of measures of angles of a triangle

$\therefore \frac{1}{2} \angle BAC + \frac{1}{2} \angle ABC + \frac{1}{2} \angle ACB = \frac{1}{2} \times \square$

....multiplying each term by $\frac{1}{2}$

$\therefore \frac{1}{2} \angle BAC + \angle PBC + \angle PCB = 90^\circ$

$\therefore \angle PBC + \angle PCB = 90^\circ - \frac{1}{2} \angle BAC$ ......(I)

In Δ BPC

$\angle BPC + \angle PBC + \angle PCB = 180^\circ$ ......sum of measures of angles of a triangle

$\therefore \angle BPC + \square = 180^\circ$ ......from (I)

$\therefore \angle BPC = 180^\circ - (90^\circ - \frac{1}{2} \angle BAC)$

$= 180^\circ - 90^\circ + \frac{1}{2} \angle BAC$

$= 90^\circ + \frac{1}{2} \angle BAC$

Practice set 3.1

1. In figure 3.8, $\angle ACD$ is an exterior angle of Δ ABC. $\angle B = 40^\circ$, $\angle A = 70^\circ$. Find the measure of $\angle ACD$.

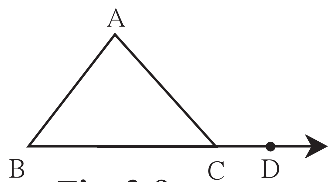


Fig. 3.8

2. In Δ PQR, $\angle P = 70^\circ$, $\angle Q = 65^\circ$ then find $\angle R$.

3. The measures of angles of a triangle are x°, $(x-20)^\circ$, $(x-40)^\circ$. Find the measure of each angle.

4. The measure of one of the angles of a triangle is twice the measure of its smallest angle and the measure of the other is thrice the measure of the smallest angle. Find the measures of the three angles.

5. In figure 3.9, measures of some angles are given. Using the measures find the values of x, y, z.

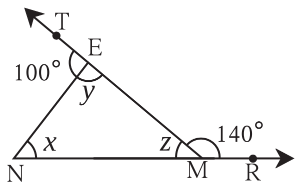


Fig. 3.9

6. In figure 3.10, line AB $||$ line DE. Find the measures of $\angle$DRE and $\angle$ARE using given measures of some angles.

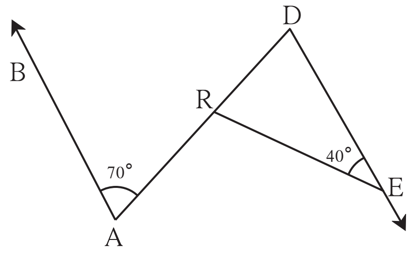


Fig. 3.10

7. In Δ ABC, bisectors of $\angle$A and $\angle$B intersect at point O. If $\angle C = 70°$. Find measure of $\angle$AOB.

8. In Figure 3.11, line AB $||$ line CD and line PQ is the transversal. Ray PT and ray QT are bisectors of $\angle$BPQ and $\angle$PQD respectively.
Prove that m$\angle PTQ = 90°$.

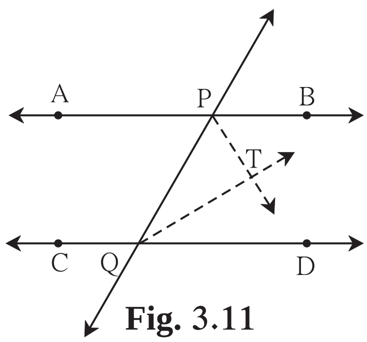


Fig. 3.11

9. Using the information in figure 3.12, find the measures of $\angle a$, $\angle b$ and $\angle c$.

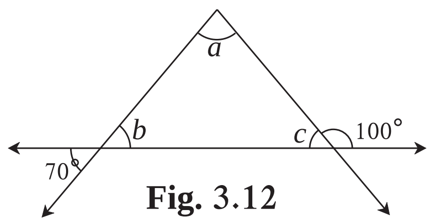


Fig. 3.12

10. In figure 3.13, line DE $||$ line GF ray EG and ray FG are bisectors of $\angle$DEF and $\angle$DFM respectively.
Prove that,
(i) $\angle DEG = \frac{1}{2} \angle EDF$ (ii) EF = FG.

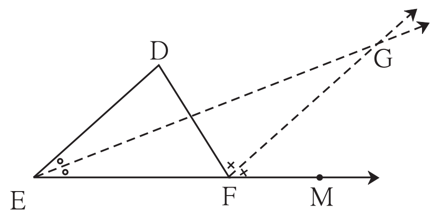


Fig. 3.13

Let's learn.

Congruence of triangles

We know that, if a segment placed upon another fits with it exactly then the two segmetns are congruent. When an angle placed upon another fits with it exactly then the two angles are congruent. Similarly, if a triangle placed upon another triangle fits exactly with it then the two triangles are said to be congruent. If Δ ABC and Δ PQR are congruent is written as $\Delta ABC \cong \Delta PQR$

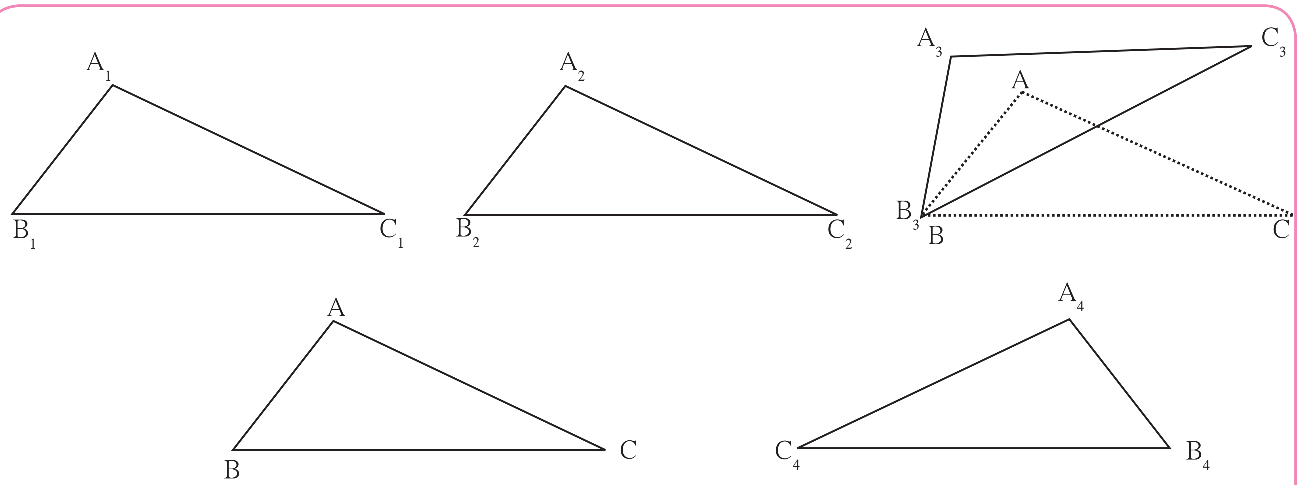


Fig. 3.14

Activity : Draw Δ ABC of any measure on a card-sheet and cut it out.

Place it on a card-sheet. Make a copy of it by drawing its border. Name it as $\Delta A_1B_1C_1$

Now slide the Δ ABC which is the cut out of a triangle to some distance and make one more copy of it. Name it $\Delta A_2B_2C_2$

Then rotate the cut out of triangle ABC a little, as shown in the figure, and make another copy of it. Name the copy as $\Delta A_3B_3C_3$. Then flip the triangle ABC, place it on another card-sheet and make a new copy of it. Name this copy as $\Delta A_4B_4C_4$.

Have you noticed that each of $\Delta A_1B_1C_1$, $\Delta A_2B_2C_2$, $\Delta A_3B_3C_3$ and $\Delta A_4B_4C_4$ is congruent with Δ ABC ? Because each of them fits exactly with Δ ABC.

Let us verify for $\Delta A_3B_3C_3$. If we place $\angle A$ upon $\angle A_3$, $\angle B$ upon $\angle B_3$ and $\angle C$ upon $\angle C_3$, then only they will fit each other and we can say that $\Delta ABC \cong \Delta A_3B_3C_3$.

We also have $AB = A_3B_3$, $BC = B_3C_3$, $CA = C_3A_3$.

Note that, while examining the congruence of two triangles, we have to write their angles and sides in a specific order, that is with a specific one-to-one correspondence.

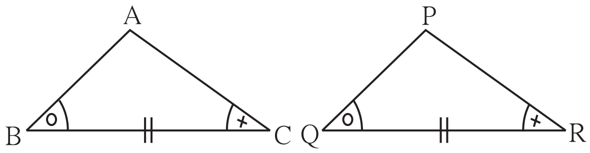
If $\Delta ABC \cong \Delta PQR$, then we get the following six equations :

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ . ... . (I) and $AB = PQ$, $BC = QR$, $CA = RP$ . . . . (II)

This means, with a one-to-one correspondence between the angles and the sides of two triangles, we get three pairs of congruent angles and three pairs of congruent sides.

Given six equations above are true for congruent triangles. For this let us see three specific equations are true then all six equations become true and hence two triangles congruent.

(1) In a correspondence, if two angles of ΔABC are equal to two angles of ΔPQR and the sides included by the respective pairs of angles are also equal, then the two triangles are congruent.



This property is called as angle-side-angle test, which in short we write A-S-A test.

Fig. 3.15

(2) In a correspondence, if two sides of Δ ABC are equal to two sides of Δ PQR and the angles included by the respective pairs of sides are also equal, then the two triangles are congruent.

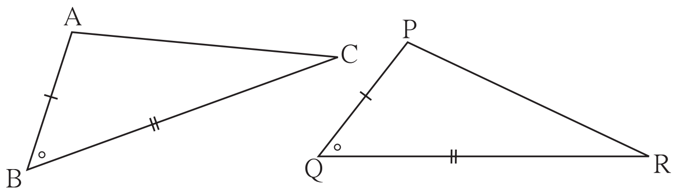


Fig. 3.16

This property is called as side-angle-side test, which in short we write S-A-S test.

(3) In a correspondence, if three sides of Δ ABC are equal to three sides of Δ PQR , then the two triangles are congruent.

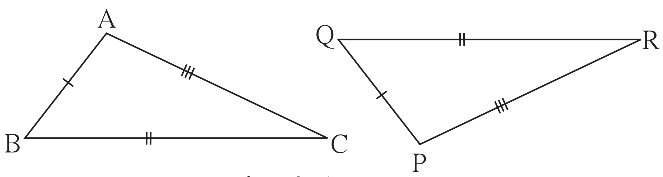


Fig. 3.17

This property is called as side-side-side test, which in short we write S-S-S test.

(4) If in Δ ABC and Δ PQR, $\angle B$ and $\angle Q$ are right angles, hypotenuses are equal and AB = PQ, then the two triangles are congruent.

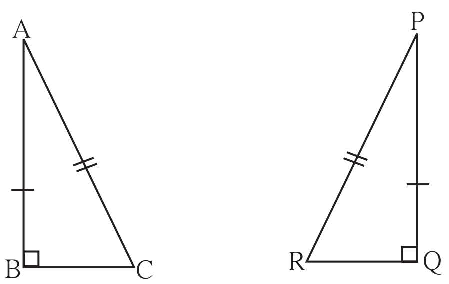


Fig. 3.18

This property is called the hypotenuse side test.

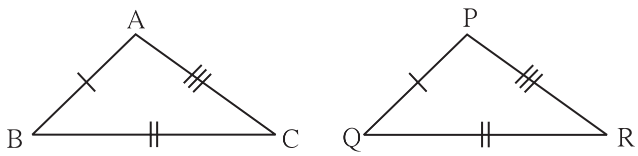
Remember this !

We have constructed triangles using the given information about parts of triangles. (For example, two angles and the included side, three sides, two sides and an included angle). We have experienced that the triangle constructed with any of these information is unique. So if by some one-to-one correspondence between two triangles, these three parts of one triangle are congruent with corresponding three parts of the other triangle then the two triangles are congruent. Then we come to know that in that correspondence their three angles and three sides are congruent. If two triangles are congruent then their respective angles and respective sides are congruent. This property is useful to solve many problems in Geometry.

Practice set 3.2

1. In each of the examples given below, a pair of triangles is shown. Equal parts of triangles in each pair are marked with the same signs. Observe the figures and state the test by which the triangles in each pair are congruent.

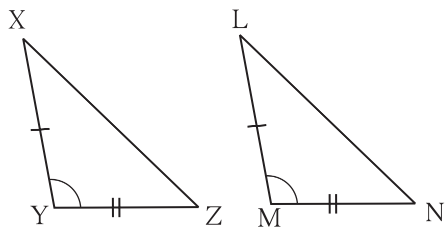
(i)



By . . . . test

Δ ABC $\cong$ Δ PQR

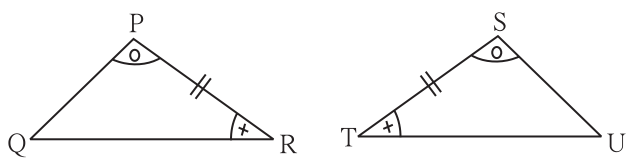
(ii)



By . . . . test

Δ XYZ $\cong$ Δ LMN

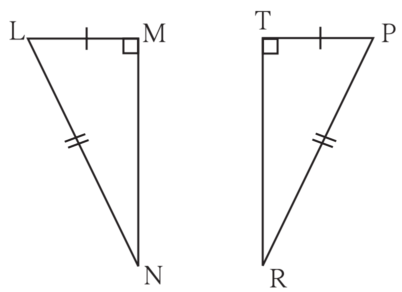
(iii)



By . . . . test

Δ PRQ $\cong$ Δ STU

(iv)



By . . . . test

Δ LMN $\cong$ Δ PTR

Fig. 3.19

2. Observe the information shown in pairs of triangles given below. State the test by which the two triangles are congruent. Write the remaining congruent parts of the triangles.

(i)

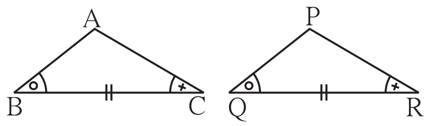


Fig. 3.20

From the information shown in the figure,
in Δ ABC and Δ PQR
$\angle$ABC $\cong$ $\angle$PQR
seg BC $\cong$ seg QR
$\angle$ACB $\cong$ $\angle$PRQ
$\therefore$ Δ ABC $\cong$ Δ PQR .... [ ] test
$\therefore$ $\angle$BAC $\cong$ [ ] ....corresponding angles of congruent triangles.

seg AB $\cong$ [ ]
and [ ] $\cong$ seg PR } ... corresponding sides of congruent triangles

(ii)

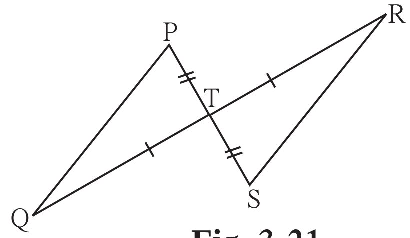


Fig. 3.21

From the information shown in the figure,,
In Δ PTQ and Δ STR
seg PT $\cong$ seg ST
$\angle$PTQ $\cong$ $\angle$STR....vertically opposite angles
seg TQ $\cong$ seg TR
$\therefore$ Δ PTQ $\cong$ Δ STR .... [ ] test
$\therefore$ $\angle$TPQ $\cong$ [ ]
and [ ] $\cong$ $\angle$TRS } ..... corresponding angles of congruent triangles.

seg PQ $\cong$ [ ] corresponding sides of congruent triangles.

3. From the information shown in the figure, state the test assuring the congruence of Δ ABC and Δ PQR. Write the remaining congruent parts of the triangles.

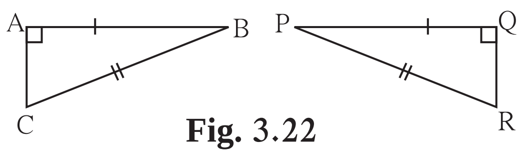


Fig. 3.22

4. As shown in the following figure, in Δ LMN and Δ PNM, LM = PN, LN = PM. Write the test which assures the congruence of the two triangles. Write their remaining congruent parts.

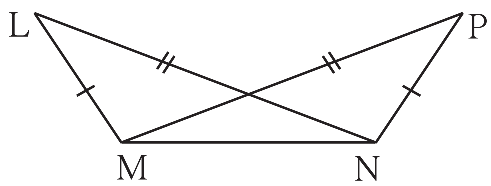


Fig. 3.23

5. In figure 3.24, seg AB $\cong$ seg CB and seg AD $\cong$ seg CD.
Prove that
Δ ABD $\cong$ Δ CBD

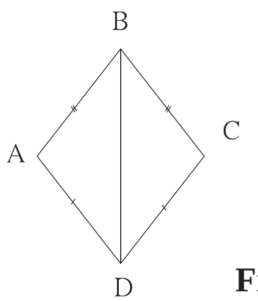


Fig. 3.24

Please note : corresponding sides of congruent triangles in short we write c.s.c.t. and corresponding angles of congruent triangles in short we write c.a.c.t.

6. In figure 3.25, $\angle P \cong \angle R$
seg PQ $\cong$ seg RQ
Prove that,
Δ PQT $\cong$ Δ RQS

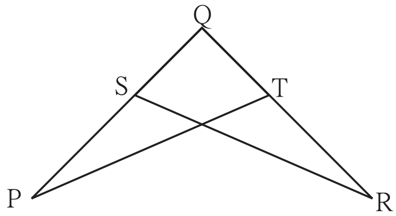


Fig. 3.25


Let's learn.

Isosceles triangle theorem

Theorem : If two sides of a triangle are congruent then the angles opposite to them are congruent.

Given : In Δ ABC, side AB $\cong$ side AC

To prove : $\angle ABC \cong \angle ACB$

Construction : Draw the bisector of $\angle BAC$ which intersects side BC at point D.

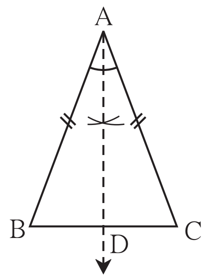


Fig. 3.26

Proof : In Δ ABD and Δ ACD

seg AB $\cong$ seg AC .... given

$\angle BAD \cong \angle CAD$........construction

seg AD $\cong$ seg AD .... common side

$\therefore$ Δ ABD $\cong$ Δ ACD ...... ____

$\therefore$ $\angle ABD \cong$ ____ .... (c.a.c.t.)

$\therefore$ $\angle ABC \cong \angle ACB$ $\because$ B - D - C

Corollary : If all sides of a triangle are congruent then its all angles are congruent. (write the proof of this corollary.)

Converse of isosceles triangle theorem

Theorem : If two angles of a triangle are congruent then the sides opposite to them are congruent.

Given : In Δ PQR, $\angle PQR \cong \angle PRQ$

To prove : Side PQ $\cong$ side PR

Construction : Draw the bisector of $\angle P$ intersecting side QR at point M

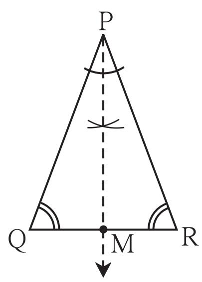


Fig. 3.27

Proof : In Δ PQM and Δ PRM

$\angle PQM \cong$ ____........ given

$\angle QPM \cong \angle RPM$........ ____

seg PM $\cong$ ____ .... common side

$\therefore$ Δ PQM $\cong$ Δ PRM ...... ____ test

$\therefore$ seg PQ $\cong$ seg PR....... c.s.c.t.

Corollary : If three angles of a triangle are congruent then its three sides also are congruent. (Write the proof of this corollary yourself.)

Both the above theorems are converses of each other also.

Similarly the corollaries of the theorems are converses of each other.

Use your brain power!

(1) Can the theorem of isosceles triangle be proved doing a different construction ?

(2) Can the theorem of isosceles triangle be proved without doing any construction ?

Let's learn.

Property of 30° - 60° - 90° triangle

Activity I

Every student in the group should draw a right angled triangle, one of the angles measuring 30°. The choice of lengths of sides should be their own. Each one should measure the length of the hypotenuse and the length of the side opposite to 30° angle.

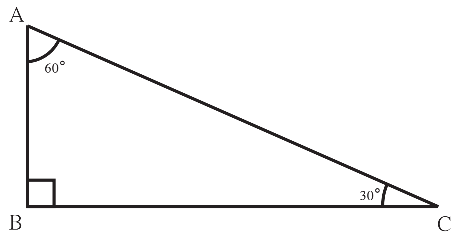


Fig. 3.28

One of the students in the group should fill in the following table.

Triangle Number	1	2	3	4
Length of the side opposite to 30° angle				
Length of the hypotenuse				

Did you notice any property of sides of right angled triangle with one of the angles measuring 30°?

Activity II

The measures of angles of a set square in your compass box are $30^\circ, 60^\circ$ and 90°. Verify the property of the sides of the set square.

Let us prove an important property revealed from these activities.

Theorem : If the acute angles of a right angled triangle have measures 30° and 60°, then the length of the side opposite to 30° angle is half the length of the hypotenuse.

(Fill in the blanks and complete the proof .)

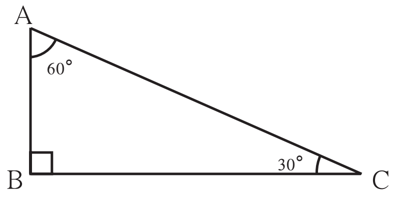


Fig. 3.29

Given : In Δ ABC

$\angle B = 90°$, $\angle C = 30°$, $\angle A = 60°$

To prove : $AB = \frac{1}{2} AC$

Construction : Take a point D on the extended seg AB such that AB = BD. Draw seg DC.

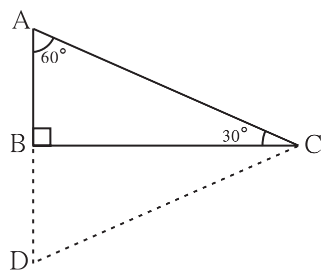


Fig. 3.30

Proof : Δ ABC and Δ DBC

seg AB $\cong$ seg DB .... $\boxed{}$

$\angle ABC \cong \angle DBC$ .... $\boxed{}$

seg BC $\cong$ seg BC .... $\boxed{}$

$\therefore$ $\Delta ABC \cong \Delta DBC$ ..... $\boxed{}$

$\therefore$ $\angle BAC \cong \angle BDC$ .... (c.a.c.t.)

In Δ ABC, $\angle BAC = 60°$ $\therefore$ $\angle BDC = 60°$

$\angle DAC = \angle ADC = \angle ACD = 60°$ ...$\because$ sum of angles of Δ ADC is 180°

$\therefore$ Δ ADC is an equilateral triangle.

$\therefore$ AC = AD = DC .... corollary of converse of isosceles triangle theorem

But $AB = \frac{1}{2}$ AD........ construction

$\therefore AB = \frac{1}{2}$ AC .... $\because$ AD = AC

Activity

With the help of the Figure 3.29 above fill in the blanks and complete the proof of the following theorem.

Theorem : If the acute angles of a right angled triangle have measures 30° and 60° then the length of the side opposite to 60° angle is $\frac{\sqrt{3}}{2} \times$ hypotenuse

Proof : In the above theorem we have proved $AB = \frac{1}{2}$ AC

$AB^2 + BC^2 = \boxed{}$ .... Pythagoras theorem

$\frac{1}{4} AC^2 + BC^2 = \boxed{}$

$\therefore BC^2 = AC^2 - \frac{1}{4} AC^2$

$\therefore BC^2 = \boxed{}$

$\therefore BC = \frac{\sqrt{3}}{2} AC$

Activity : Complete the proof of the theorem.

Theorem : If measures of angles of a triangle are 45°, 45°, 90° then the length of each side containing the right angle is $\frac{1}{\sqrt{2}} \times$ hypotenuse.

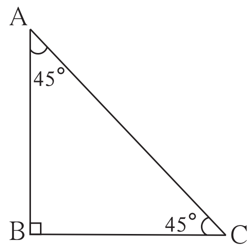


Fig. 3.31

Proof : In Δ ABC, $\angle B = 90^\circ$ and $\angle A = \angle C = 45^\circ$

$$\therefore BC = AB$$

By Pythagoras theorem

$$AB^2 + BC^2 = \boxed{}$$

$$AB^2 + \boxed{} = AC^2 \quad \ldots \because (BC = AB)$$

$$\therefore 2AB^2 = \boxed{}$$

$$\therefore AB^2 = \boxed{}$$

$$\therefore AB = \frac{1}{\sqrt{2}} AC$$

This property is called 45°- 45°- 90° theorem.

Remember this !

(1) If the acute angles of a right angled triangle are 30°, 60° then the length of side opposite to 30° angle is half of hypotenuse and the length of side opposite to 60° angle is $\frac{\sqrt{3}}{2}$ hypotenuse . This property is called $30^\circ - 60^\circ - 90^\circ$ theorem.

(2) If acute angles of a right angled triangle are 45°, 45° then the length of each side containing the right angle is $\frac{\text{hypotenuse}}{\sqrt{2}}$.

This property is called $45^\circ - 45^\circ - 90^\circ$ theorem

Let's recall.

Median of a triangle

The segment joining a vertex and the mid-point of the side opposite to it is called a **Median** of the triangle.

In Figure 3.32, point D is the mid point of side BC.

$\therefore$ seg AD is a median of Δ ABC.

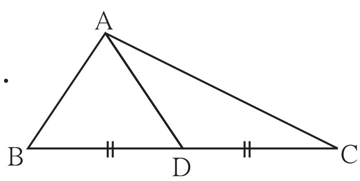


Fig. 3.32

Activity I : Draw a triangle ABC. Draw medians AD, BE and CF of the triangle. Let their point of concurrence be G, which is called the centroid of the triangle. Compare the lengths of AG and GD with a divider. Verify that the length of AG is twice the length of GD. Similarly, verify that the length of BG is twice the length of GE and the length of CG is twice the length of GF. Hence note the following property of medians of a triangle.

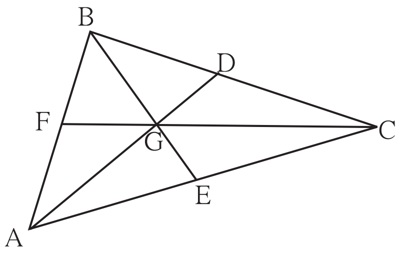


Fig. 3.33

The point of concurrence of medians of a triangle divides each median in the ratio 2 : 1.

Activity II : Draw a triangle ABC on a card board. Draw its medians and denote their point of concurrence as G. Cut out the triangle.

Now take a pencil. Try to balance the triangle on the flat tip of the pencil. The triangle is balanced only when the point G is on the flat tip of the pencil.

This activity shows an important property of the **centroid** (point of concurrence of the medians) of the triangle.

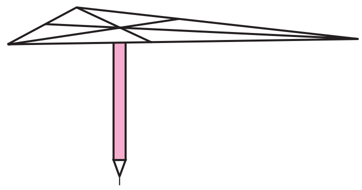


Fig. 3.34

Let's learn.

Property of median drawn on the hypotenuse of right triangle

Activity : In the figure 3.35, Δ ABC is a right angled triangle. seg BD is the median on hypotenuse.

Measure the lengths of the following segments

AD =......... DC =............ BD = ...

From the measurements verify that BD = $\frac{1}{2}$ AC .

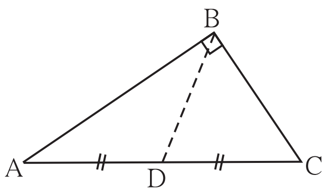


Fig. 3.35

Now let us prove the property, the length of the median is half the length of the hypotenuse.

Theorem : In a right angled triangle, the length of the median of the hypotenuse is half the length of the hypotenuse.

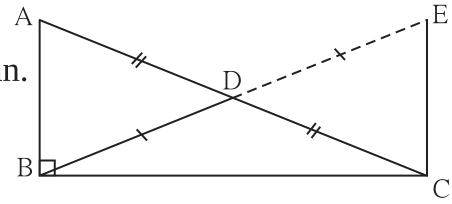


Fig. 3.36

Given : In Δ ABC, $\angle B = 90°$, seg BD is the median.

To prove : BD = $\frac{1}{2}$ AC

Construction : Take point E on the ray BD such that B - D - E and l(BD) = l(DE). Draw seg EC.

Proof : (Main steps are given. Write the steps in between with reasons and complete the proof.)

Δ ADB $\cong$ Δ CDE .... by S-A-S test

line AB $||$ line EC ....by test of alternate angles

Δ ABC $\cong$ Δ ECB .... by S-A-S test

BD = $\frac{1}{2}$ AC

Remember this !

In a right angled triangle, the length of the median on its hypotenuse is half the length of the hypotenuse.

Practice set 3.3

1. Find the values of x and y using the information shown in figure 3.37. Find the measure of $\angle$ABD and m$\angle$ACD.

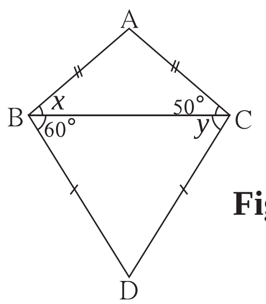


Fig. 3.37

2. The length of hypotenuse of a right angled triangle is 15. Find the length of median of its hypotenuse.

3. In Δ PQR, $\angle Q = 90°$, PQ = 12, QR = 5 and QS is a median. Find l(QS).

4. In figure 3.38, point G is the point of concurrence of the medians of Δ PQR . If GT = 2.5, find the lengths of PG and PT.

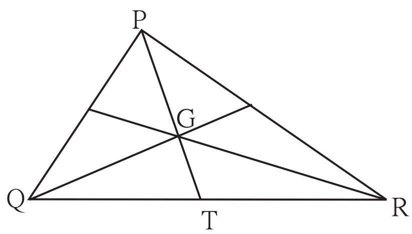


Fig. 3.38

Let's recall.

Activity : Draw a segment AB of convenient length. Lebel its mid-point as M. Draw a line *l* passing through the point M and perpendicular to seg AB.

Did you notice that the line *l* is the perpendicular bisector of seg AB ?

Now take a point P anywhere on line *l*. Compare the distance PA and PB with a divider. What did you find ? You should have noticed that PA = PB. This observation shows that any point on the perpendicular bisector of a segment is equidistant from its end points.

Now with the help of a compass take any two points like C and D, which are equidistant from A and B. Did all such points lie on the line *l* ? What did you notice from the observation ? Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

These two properties are two parts of the perpendicular bisector theorem. Let us now prove them.

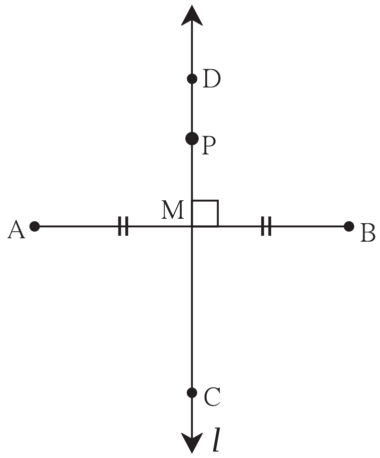


Fig. 3.39

Let's learn.

Perpendicular bisector theorem

Part I : Every point on the perpendicular bisector of a segment is equidistant from the end points of the segment.

Given : line *l* is the perpendicular bisector of seg AB at point M.

Point P is any point on *l*,

To prove : PA = PB

Construction : Draw seg AP and seg BP.

Proof : In Δ PMA and Δ PMB

seg PM $\cong$ seg PM .... common side

$\angle$PMA $\cong$ $\angle$PMB ....each is a right angle

seg AM $\cong$ seg BM ....given

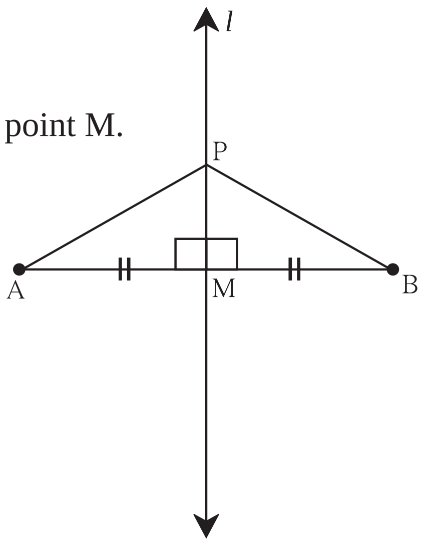


Fig. 3.40

$\therefore$ Δ PMA $\cong$ Δ PMB ...... S-A-S test
$\therefore$ seg PA $\cong$ seg PB ....c.s.c.t.
$\therefore$ l (PA) = l (PB)
Hence every point on the perpendicular bisector of a segment is equidistant from the end points of the segment.

Part II : Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

Given : Point P is any point equidistant from the end points of seg AB. That is, PA = PB.

To prove : Point P is on the perpendicular bisector of seg AB.

Construction : Take mid-point M of seg AB and draw line PM.

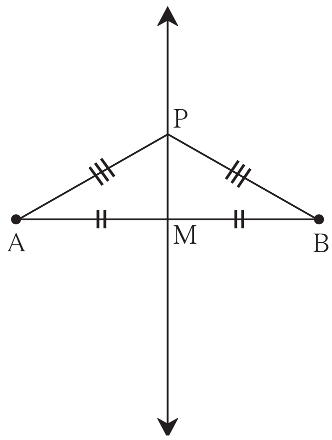


Fig. 3.41

Proof : In Δ PAM and Δ PBM

seg PA $\cong$ seg PB .... [ ]

seg AM $\cong$ seg BM .... [ ]

seg PM $\cong$ [ ] .... common side

$\therefore$ Δ PAM $\cong$ Δ PBM ...... [ ] test.

$\therefore$ $\angle$PMA $\cong$ $\angle$PMB.......c.a.c.t.

But $\angle$PMA + [ ] = 180°

$\angle$PMA + $\angle$PMA = 180° .... ($\because$ $\angle$PMB = $\angle$PMA)

2 $\angle$PMA = [ ]

$\therefore$ $\angle$PMA = 90°

$\therefore$ seg PM $\perp$ seg AB ......(1)

But Point M is the midpoint of seg AB. ......construction .... (2)

$\therefore$ line PM is the perpendicular bisector of seg AB. So point P is on the perpendicular bisector of seg AB

Angle bisector theorem

Part I : Every point on the bisector of an angle is equidistant from the sides of the angle.

Given : Ray QS is the bisector of $\angle$PQR .
Point A is any point on ray QS
seg AB $\perp$ ray QP seg AC $\perp$ ray QR

To prove : seg AB $\cong$ seg AC

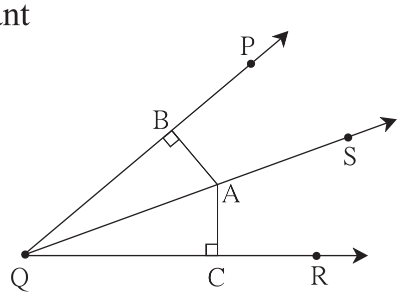


Fig. 3.42

Proof : Write the proof using test of congruence of triangles.

Part II : Any point equidistant from sides of an angle is on the bisector of the angle.

Given : A is a point in the interior of ∠PQR.

seg AC ⊥ ray QR seg AB ⊥ ray QP

and AB = AC

To prove : Ray QA is the bisector of ∠PQR.

That is ∠BQA = ∠CQA

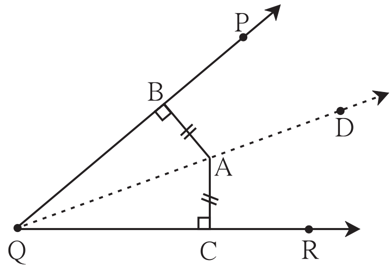


Fig. 3.43

Proof : Write the proof using proper test of congruence of triangles.

Let’s recall.

Activity

As shown in the figure, draw Δ XYZ such that XZ > side XY

Find which of ∠Z and ∠Y is greater.

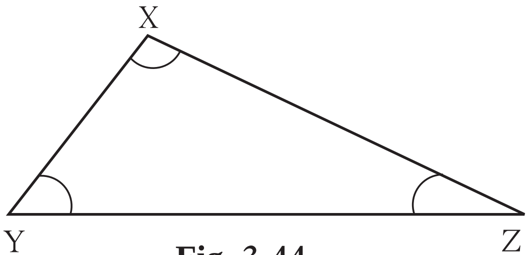


Fig. 3.44

Let’s learn.

Properties of inequalities of sides and angles of a triangle

Theorem : If two sides of a triangle are unequal, then the angle opposite to the greater side is greater than angle opposite to the smaller side.

Given : In Δ XYZ, side XZ > side XY

To prove : ∠XYZ > ∠XZY

Construction : Take point P on side XZ such that XY = XP, Draw seg YP.

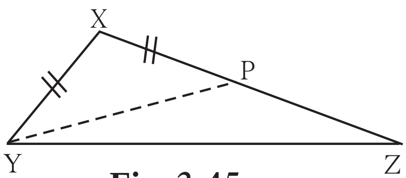


Fig. 3.45

Proof : In Δ XYP

XY = XP ....construction

∴ ∠XYP = ∠XPY.....isosceles triangle theorem .....(I)

∠XPY is an exterior angle of Δ YPZ.

∴ ∠XPY > ∠PZY ....exterior angle theorem

∴ ∠XYP > ∠PZY ....from (I)

∴ ∠XYP + ∠PYZ > ∠PZY ....If a > b and c > 0 then a + c > b

∴ ∠XYZ > ∠PZY, that is ∠XYZ > ∠XZY

Theorem : If two angles of a triangle are unequal then the side opposite to the greater angle is greater than the side opposite to smaller angle.

The theorem can be proved by indirect proof. Complete the following proof by filling in the blanks.

Given : In ΔABC, $\angle B > \angle C$

To prove : $AC > AB$

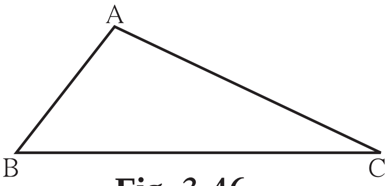


Fig. 3.46

Proof : There are only three possibilities regarding the lengths of side AB and side AC of Δ ABC

(i) $AC < AB$ (ii) ______ (iii) ______

(i) Let us assume that $AC < AB$.

If two sides of a triangle are unequal then the angle opposite to greater side is ______ .

$\therefore$ $\angle C >$ ______

But $\angle C < \angle B$ .... (given)

This creates a contradiction.

$\therefore$ ______ $<$ ______ is wrong.

(ii) If $AC = AB$

then $\angle B = \angle C$

But ______ $>$ ______ ...... (given)

This also creates a contradiction.

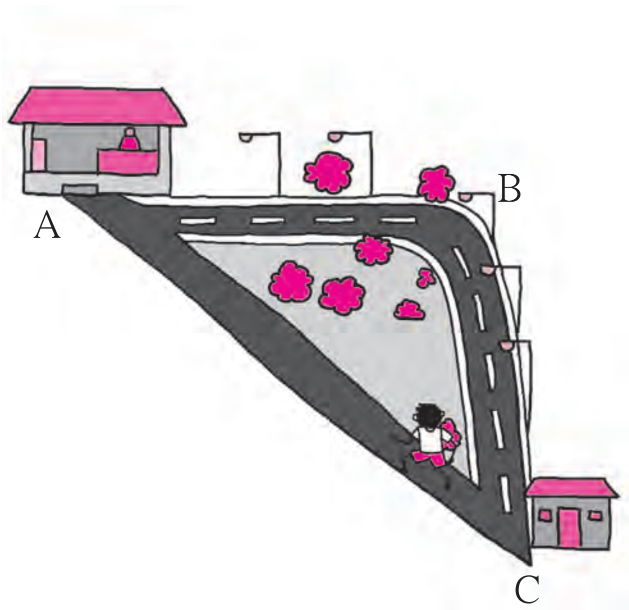
$\therefore$ ______ $=$ ______ is wrong

$\therefore$ $AC > AB$ is the only remaining possibility.

$\therefore$ $AC > AB$

Let's recall.

As shown in the adjacent picture, there is a shop at A. Sameer was standing at C. To reach the shop, he choose the way $C \rightarrow A$ instead of $C \rightarrow B \rightarrow A$, because he knew that the way $C \rightarrow A$ was shorter than the way $C \rightarrow B \rightarrow A$. So which property of a triangle had he realised ?



The sum of two sides of a triangle is greater than its third side.

Let us now prove the property.

Theorem : The sum of any two sides of a triangle is greater than the third side.

Given : Δ ABC is any triangle.

To prove : AB + AC > BC
AB + BC > AC
AC + BC > AB

Construction : Take a point D on ray BA such that AD = AC.

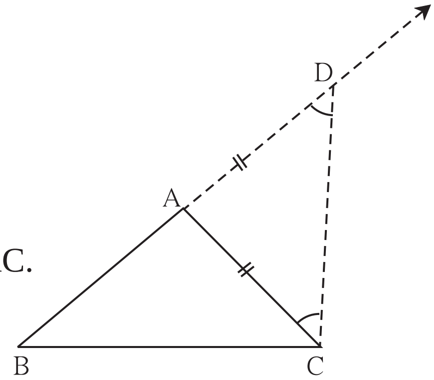


Fig. 3.47

Proof : In Δ ACD, AC = AD ..... construction

$\therefore$ $\angle$ACD = $\angle$ADC ...... c.a.c.t.

$\therefore$ $\angle$ACD + $\angle$ACB > $\angle$ADC

$\therefore$ $\angle$BCD > $\angle$ADC

$\therefore$ side BD > side BC ....the side opposite to greater angle is greater

$\therefore$ BA + AD > BC ....$\because$ BD = BA + AD

BA + AC > BC .... $\because$ AD = AC

Similarly we can prove that AB + BC > AC

and BC + AC > AB.

Practice set 3.4

1. In figure 3.48, point A is on the bisector of $\angle$XYZ.
If AX = 2 cm then find AZ.

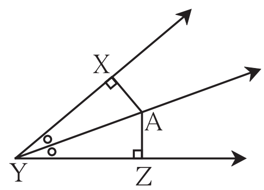


Fig. 3.48

2.

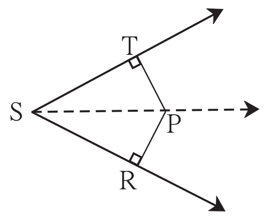


Fig. 3.49

In figure 3.49, $\angle$RST = 56°, seg PT $\perp$ ray ST, seg PR $\perp$ ray SR and seg PR $\cong$ seg PT
Find the measure of $\angle$RSP.
State the reason for your answer.

3. In Δ PQR, PQ = 10 cm, QR = 12 cm, PR = 8 cm. Find out the greatest and the smallest angle of the triangle.

4. In Δ FAN, $\angle$F = 80°, $\angle$A = 40° . Find out the greatest and the smallest side of the triangle. State the reason.

5. Prove that an equilateral triangle is equiangular.

6. Prove that, if the bisector of $\angle$BAC of Δ ABC is perpendicular to side BC, then Δ ABC is an isosceles triangle.

7. In figure 3.50, if seg PR $\cong$ seg PQ, show that seg PS > seg PQ.

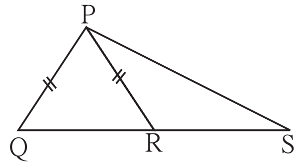


Fig. 3.50

8. In figure 3.51, in Δ ABC, seg AD and seg BE are altitudes and AE = BD.

Prove that seg AD $\cong$ seg BE

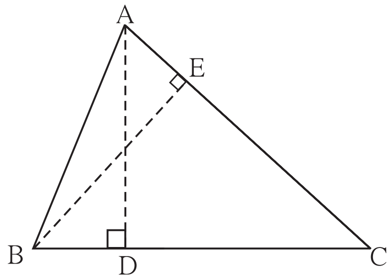
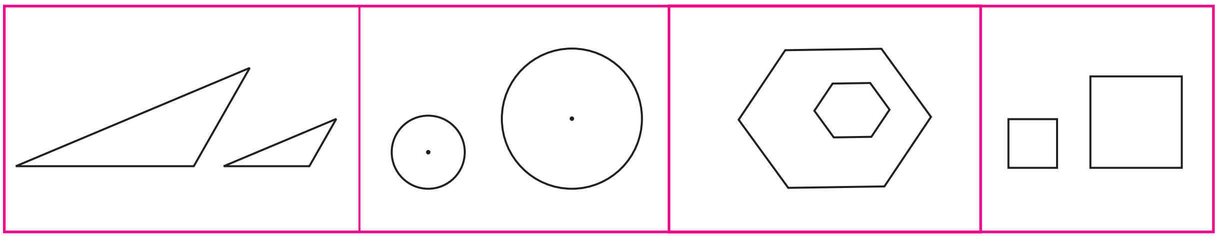


Fig. 3.51

Let's learn.

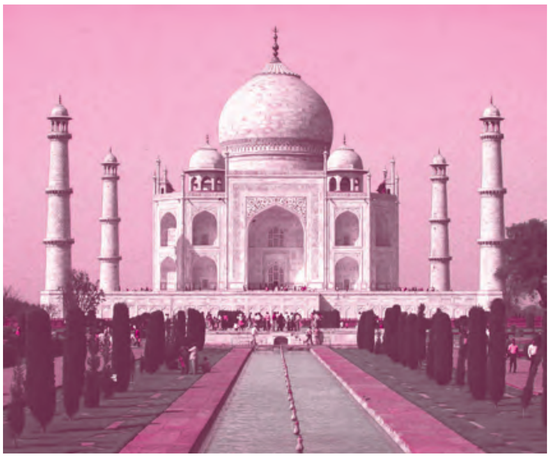
Similar triangles

Observe the following figures.



The pairs of figures shown in each part have the same shape but their sizes are different. It means that they are not congruent.

Such like looking figures are called similar figures.



We find similarity in a photo and its enlargement, also we find similarity between a road-map and the roads.

The proportionality of all sides is an important property of similarity of two figures. But the angles in the figures have to be of the same measure. If the angle between this roads is not the same in its map, then the map will be misleading.

ICT Tools or Links

Take a photograph on a mobile or a computer. Recall what you do to reduce it or to enlarge it. Also recall what you do to see a part of the photograph in detail.

Now we shall learn properties of similar triangles through an activity.

Activity : On a card-sheet, draw a triangle of sides 4 cm, 3 cm and 2 cm. Cut it out. Make 13 more copies of the triangle and cut them out from the card sheet.
Note that all these triangular pieces are congruent. Arrange them as shown in the following figure and make three triangles out of them.

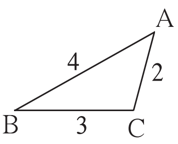


Fig. 3.52

1 triangle

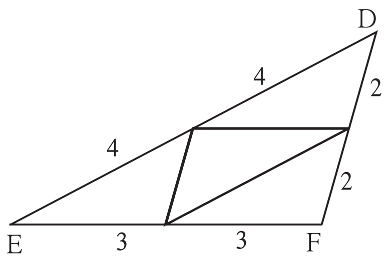


Fig. 3.53

4 triangles

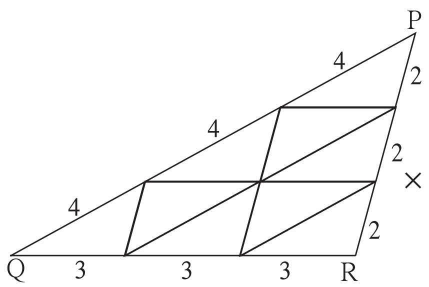


Fig. 3.54

9 triangles

Δ ABC and Δ DEF are similar in the correspondence ABC $\leftrightarrow$ DEF.

$\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$

and $\frac{AB}{DE} = \frac{4}{8} = \frac{1}{2}$; $\frac{BC}{EF} = \frac{3}{6} = \frac{1}{2}$; $\frac{AC}{DF} = \frac{2}{4} = \frac{1}{2}$,

........the corresponding sides are in proportion.

Similarly, consider Δ DEF and Δ PQR. Are their angles congruent and sides proportional in the correspondence DEF $\leftrightarrow$ PQR ?

Let's learn.

Similarity of triangles

In Δ ABC and Δ PQR, If (i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and

(ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$; then Δ ABC and Δ PQR are called similar triangles.

'Δ ABC and Δ PQR are similar' is written as '$\Delta ABC \sim \Delta PQR$'.

Let us learn the relation between the corresponding angles and corresponding sides of similar triangles through an activity.

Activity : Draw a triangle $\Delta A_1B_1C_1$ on a card-sheet and cut it out. Measure $\angle A_1, \angle B_1, \angle C_1$.

Draw two more triangles $\Delta A_2B_2C_2$ and $\Delta A_3B_3C_3$ such that

$\angle A_1 = \angle A_2 = \angle A_3$, $\angle B_1 = \angle B_2 = \angle B_3$, $\angle C_1 = \angle C_2 = \angle C_3$

and $B_1C_1 > B_2C_2 > B_3C_3$. Now cut these two triangles also. Measure the lengths of the three triangles. Arrange the triangles in two ways as shown in the figure.

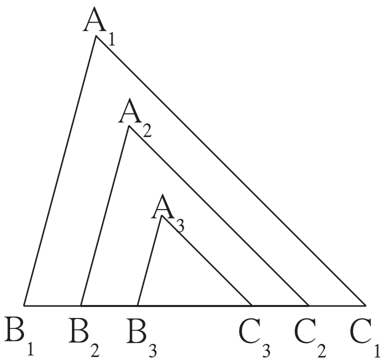


Fig. 3.55

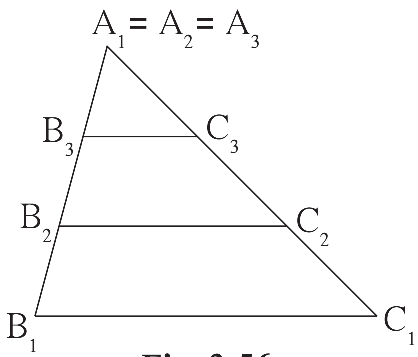


Fig. 3.56

Check the ratios $\frac{A_1B_1}{A_2B_2}, \frac{B_1C_1}{B_2C_2}, \frac{A_1C_1}{A_2C_2}$. You will notice that the ratios are equal.

Similarly, see whether the ratios $\frac{A_1C_1}{A_3C_3}, \frac{B_1C_1}{B_3C_3}, \frac{A_1B_1}{A_3B_3}$ are equal.

From this activity note that, when corresponding angles of two triangles are equal, the ratios of their corresponding sides are also equal. That is, their corresponding sides are in the same proportion.

We have seen that, in Δ ABC and Δ PQR if

(i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$, then (ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

This means, if corresponding angles of two triangles are equal then the corresponding sides are in the same proportion.

This rule can be proved elaborately. We shall use it to solve problems.

Remember this !

- If corresponding angles of two triangles are equal then the two triangles are similar.
- If two triangles are similar then their corresponding sides are in proportion and corresponding angles are congruent.

Ex. Some information is shown in Δ ABC and Δ PQR in figure 3.57. Observe it. Hence find the lengths of side AC and PQ.

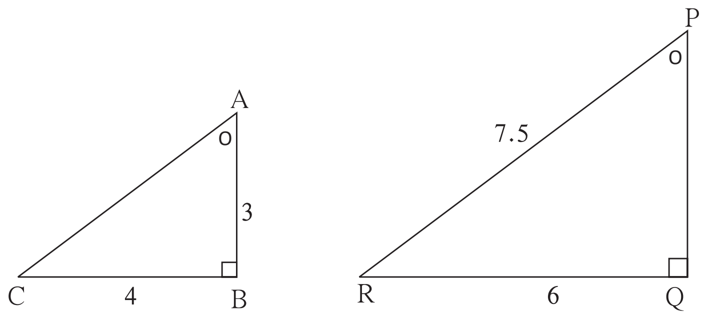


Fig. 3.57

Solution : The sum of all angles of a triangle is 180°.

It is given that,

$\angle A = \angle P$ and $\angle B = \angle Q$ $\therefore$ $\angle C = \angle R$

$\therefore$ Δ ABC and Δ PQR are equiangular triangles.

$\therefore$ there sides are propotional.

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\therefore \frac{3}{PQ} = \frac{4}{6} = \frac{AC}{7.5}$$

$$\therefore 4 \times PQ = 18$$

$$\therefore PQ = \frac{18}{4} = 4.5$$

Similarly $6 \times AC = 7.5 \times 4$

$$\therefore AC = \frac{7.5 \times 4}{6} = \frac{30}{6} = 5$$

Practice set 3.5

1. If Δ XYZ ~ Δ LMN, write the corresponding angles of the two triangles and also write the ratios of corresponding sides.
2. In Δ XYZ, XY = 4 cm, YZ = 6 cm, XZ = 5 cm, If Δ XYZ ~ Δ PQR and PQ = 8 cm then find the lengths of remaining sides of Δ PQR.
3. Draw a sketch of a pair of similar triangles. Label them. Show their corresponding angles by the same signs. Show the lengths of corresponding sides by numbers in proportion.

Let's recall.

While preparing a map of a locality, you have to show the distances between different spots on roads with a proper scale. For example, 1 cm = 100 m, 1 cm = 50 m etc. Did you think of the properties of triangle ? Keep in mind that side opposite to greater angle is greater.

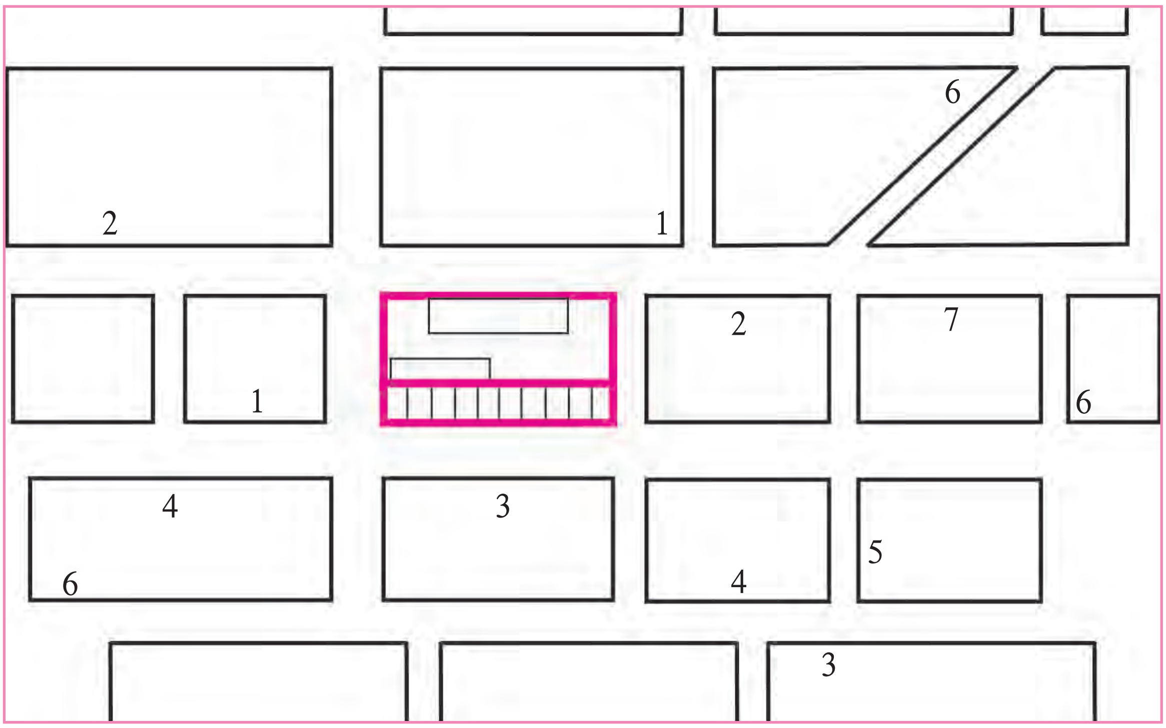
Project :

Prepare a map of road surrounding your school or home, upto a distance of about 500 metre.

How will you measure the distance between two spots on a road ?

While walking, count how many steps cover a distance of about two metre. Suppose, your three steps cover a distance of 2 metre. Considering this proportion 90 steps means 60 metre. In this way you can judge the distances between different spots on roads and also the lengths of roads. You have to judge the measures of angles also where two roads meet each other. Choosing a proper scale for lengths of roads, prepare a map. Try to show shops, buildings, bus stops, rickshaw stand etc. in the map.

A sample map with legend is given below



Legend: 1. Book store 2. Bus stop 3. Stationery shop 4. Bank
5. Medical store 6. Restaurant 7. Cycle shop

Problem set 3

1. Choose the correct alternative answer for the following questions.

(i) If two sides of a triangle are 5 cm and 1.5 cm, the lenght of its third side cannot be . . . .

(A) 3.7 cm (B) 4.1 cm (C) 3.8 cm (D) 3.4 cm

(ii) In Δ PQR, If $\angle R > \angle Q$ then . . . .

(A) $QR > PR$ (B) $PQ > PR$ (C) $PQ < PR$ (D) $QR < PR$

(iii) In Δ TPQ, $\angle T = 65^\circ$, $\angle P = 95^\circ$ which of the following is a true statement ?

(A) $PQ < TP$ (B) $PQ < TQ$ (C) $TQ < TP < PQ$ (D) $PQ < TP < TQ$

2. Δ ABC is isosceles in which AB = AC. Seg BD and seg CE are medians. Show that BD = CE.

3. In Δ PQR, If $PQ > PR$ and bisectors of $\angle Q$ and $\angle R$ intersect at S. Show that $SQ > SR$.

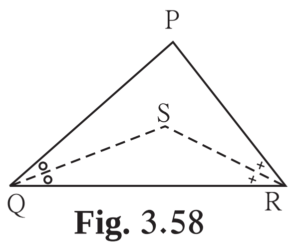


Fig. 3.58

4. In figure 3.59, point D and E are on side BC of Δ ABC, such that BD = CE and AD = AE. Show that $\Delta ABD \cong \Delta ACE$.

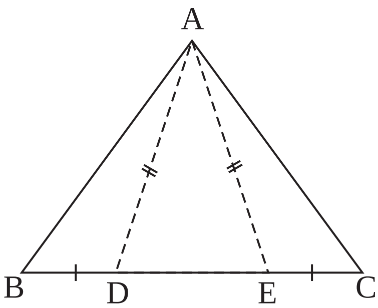


Fig. 3.59

5. In figure 3.60, point S is any point on side QR of Δ PQR

Prove that : $PQ + QR + RP > 2PS$

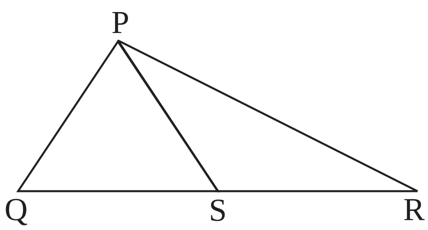


Fig. 3.60

6. In figure 3.61, bisector of $\angle BAC$ intersects side BC at point D. Prove that AB > BD

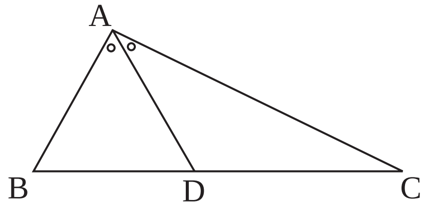


Fig. 3.61

7. In figure 3.62, seg PT is the bisector of $\angle QPR$. A line through R intersects ray QP at point S. Prove that PS = PR

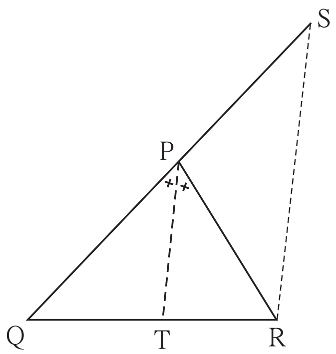


Fig. 3.62

8. In figure 3.63, seg AD $\perp$ seg BC. seg AE is the bisector of $\angle CAB$ and C - E - D.
Prove that

$$\angle DAE = \frac{1}{2}(\angle C - \angle B)$$

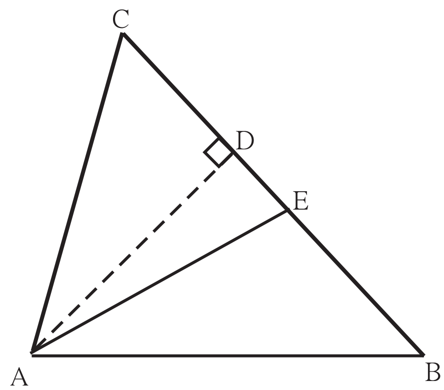


Fig. 3.63

Use your brain power!

We have learnt that if two triangles are equiangular then their sides are in proportion. What do you think if two quadrilaterals are equiangular ? Are their sides in proportion? Draw different figures and verify.

Verify the same for other polygons.

4 Constructions of Triangles

23M8J3

Let's study.

To construct a triangle, if following information is given.

- **Base, an angle adjacent to the base and sum of lengths of two remaining sides.**
- **Base, an angle adjacent to the base and difference of lengths of remaining two sides.**
- **Perimeter and angles adjacent to the base.**

Let's recall.

In previous standard we have learnt the following triangle constructions.

* To construct a triangle when its three sides are given.
* To construct a triangle when its base and two adjacent angles are given.
* To construct a triangle when two sides and the included angle are given.
* To construct a right angled triangle when its hypotenuse and one side is given.

Perpendicular bisector Theorem

- Every point on the perpendicular bisector of a segment is equidistant from its end points.
- Every point equidistant from the end points of a segment is on the perpendicular bisector of the segment.

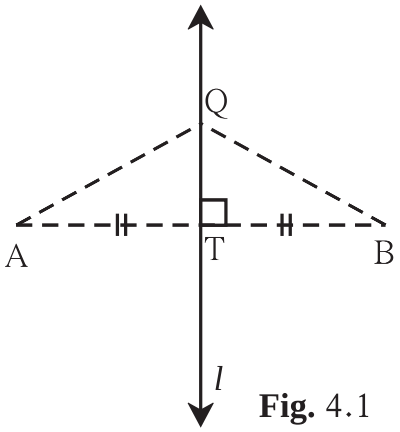


Fig. 4.1

Let's learn.

Constructions of triangles

To construct a triangle, three conditions are required. Out of three sides and three angles of a triangle two parts and some additional information about them is given, then we can construct a triangle using them.

We frequently use the following property in constructions.

If a point is on two different lines then it is the intersecrtion of the two lines.

Construction I

To construct a triangle when its base, an angle adjacent to the base and the sum of the lengths of remaining sides is given.

Ex. Construct Δ ABC in which BC = 6.3 cm, $\angle$B = 75° and AB + AC = 9 cm.

Solution : Let us first draw a rough figure of expected triangle.

Explanation : As shown in the rough figure, first we draw seg BC = 6.3 cm of length. On the ray making an angle of 75° with seg BC, mark point D such that

BD = AB + AC = 9 cm

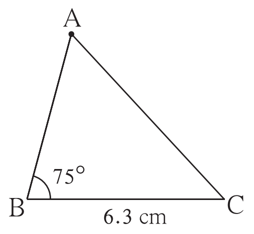
Now we have to locate point A on ray BD.

BA + AD = BA + AC = 9

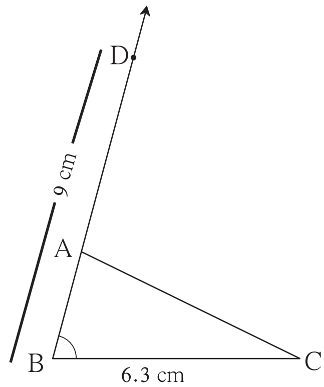
$\therefore$ AD = AC

$\therefore$ point A is on the perpendicular bisector of seg CD.

$\therefore$ the point of intersection of ray BD and the perpendicular bisector of seg CD is point A.



Rough figure 4.2



Rough figure 4.3

Steps of construction

(1) Draw seg BC of length 6.3 cm.

(2) Draw ray BP such that m$\angle$ PBC = 75°.

(3) Mark point D on ray BP such that d(B,D) = 9 cm

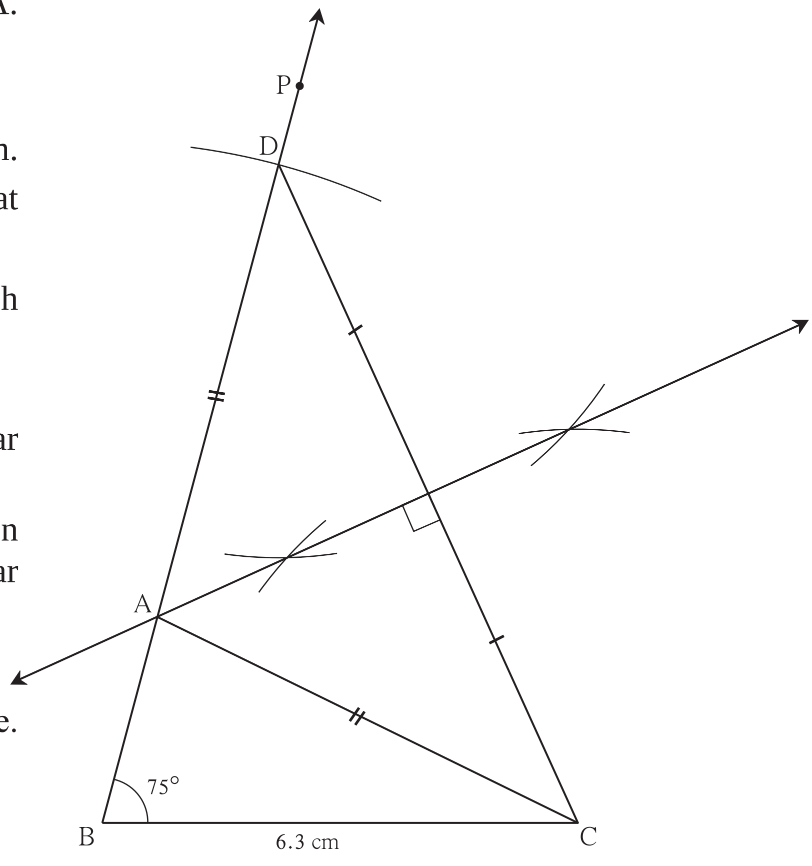
(4) Draw seg DC.

(5) Construct the perpendicular bisector of seg DC .

(6) Name the point of intersection of ray BP and the perpendicular bisector of CD as A.

(7) Draw seg AC.

Δ ABC is the required triangle.



Fair fig. 4.4

Practice set 4.1

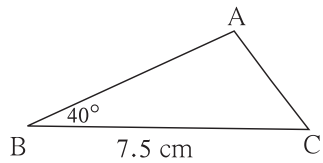
1. Construct Δ PQR, in which QR = 4.2 cm, m$\angle$Q = 40° and PQ + PR = 8.5 cm
2. Construct Δ XYZ, in which YZ = 6 cm, XY + XZ = 9 cm. $\angle$XYZ = 50°
3. Construct Δ ABC, in which BC = 6.2 cm, $\angle$ACB = 50°, AB + AC = 9.8 cm
4. Construct Δ ABC, in which BC = 3.2 cm, $\angle$ACB = 45° and perimeter of Δ ABC is 10 cm

Construction II

To construct a triangle when its base, angle adjacent to the base and difference between the remaining sides is given.

Ex (1) Construct Δ ABC, such that BC = 7.5 cm, $\angle$ABC = 40°, AB – AC = 3 cm.

Solution : Let us draw a rough figure.



Rough Figure 4.5

Explanation : AB – AC = 3 cm $\therefore$ AB > AC

Draw seg BC. We can draw the ray BL such that $\angle$LBC = 40°. We have to locate point A on ray BL. Take point D on ray BL such that BD = 3 cm.

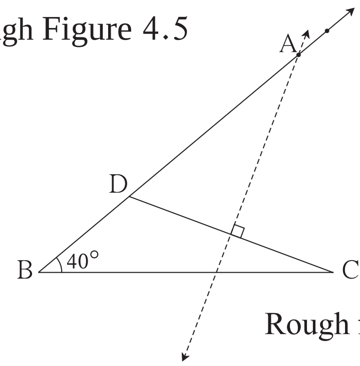
Now , B-D-A and BD = AB – AD = 3.

It is given that AB – AC = 3

$\therefore$ AD = AC

$\therefore$ point A is on the perpendicular bisector of seg DC.

$\therefore$ point A is the intersection of ray BL and the perpendicular bisector of seg DC.



Rough figure 4.6

Steps of construction

(1) Draw seg BC of length7.5 cm.

(2) Draw ray BL such that $\angle$LBC = 40°

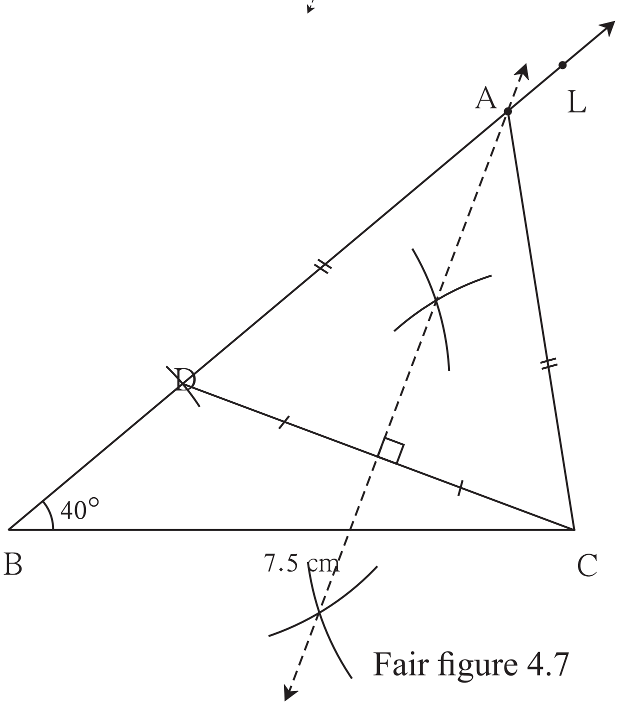
(3) Take point D on ray BL such that BD = 3 cm.

(4) Construct the perpendicular bisector of seg CD.

(5) Name the point of intersection of ray BL and the perpendicular bisector of seg CD as A.

(6) Draw seg AC.

Δ ABC is required triangle.



Fair figure 4.7

Ex. 2 Construct Δ ABC, in which side BC = 7 cm, $\angle$B = 40° and AC – AB = 3 cm.

Solution : Let us draw a rought figure.

seg BC = 7 cm. AC > AB.

We can draw ray BT such that

$\angle$ TBC = 40°

Point A is on ray BT. Take point D on opposite ray of ray BT such that

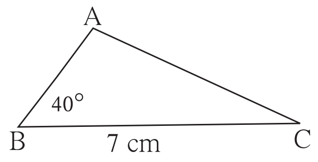
BD = 3 cm.

Now AD = AB + BD = AB + 3 = AC

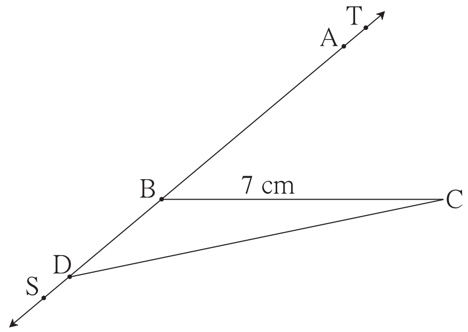
($\because$ AC – AB = 3 cm.)

$\therefore$ AD = AC

$\therefore$ point A is on the perpendicular bisector of seg CD.



Rough figure 4.8



Rough figure 4.9

Steps of construction

(1) Draw BC of length 7 cm.

(2) Draw ray BT such that $\angle$ TBC = 40°

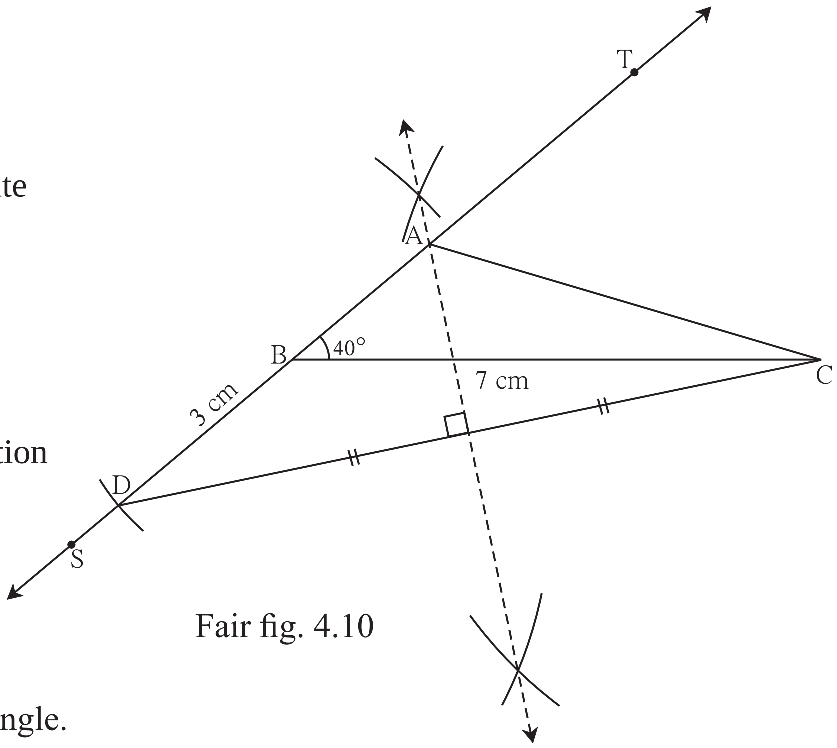
(3) Take point D on the opposite ray BS of ray BT such that BD = 3 cm.

(4) Construct perpendicular bisector of seg DC.

(5) Name the point of intersection of ray BT and the perpendicular bisector of DC as A.

(6) Draw seg AC.

Δ ABC is the required triangle.



Fair fig. 4.10

Practice set 4.2

1. Construct Δ XYZ, such that YZ = 7.4 cm, $\angle$XYZ = 45° and XY – XZ = 2.7 cm.

2. Construct Δ PQR, such that QR = 6.5 cm, $\angle$PQR = 60° and PQ – PR = 2.5 cm.

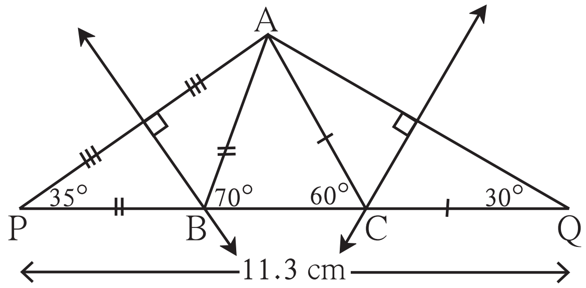
3. Construct Δ ABC, such that BC = 6 cm, $\angle$ABC = 100° and AC – AB = 2.5 cm.

Construction III

To construct a triangle, if its perimeter, base and the angles which include the base are given.

Ex. Construct Δ ABC such that AB + BC + CA = 11.3 cm, $\angle B = 70^\circ$, $\angle C = 60^\circ$.

Solution : Let us draw a rough figure.



Rough Fig. 4.11

Explanation : As shown in the figure, points P and Q are taken on line BC such that,

PB = AB, CQ = AC

$\therefore$ PQ = PB + BC + CQ = AB + BC +AC = 11.3 cm.

Now in ΔPBA, PB = BA

$\therefore$ $\angle APB = \angle PAB$ and $\angle APB + \angle PAB$ = extieror angleABC = 70°

......theorem of remote interior angles

$\therefore$ $\angle APB = \angle PAB = 35^\circ$ Similarly, $\angle CQA = \angle CAQ = 30^\circ$

Now we can draw Δ PAQ, as its two angles and the included side is known.

Since BA = BP , point B lies on the perpendicular bisector of seg AP.

Similarly, CA = CQ, therefore point C lies on the perpendicular bisector of seg AQ

$\therefore$ by constructing the perpendicular bisectors of seg AP and AQ we can get points B and C, where the perpendicular bisectors intersect line PQ.

Steps of construction

(1) Draw seg PQ of 11.3 cm length.

(2) Draw a ray making angle of 35° at point P.

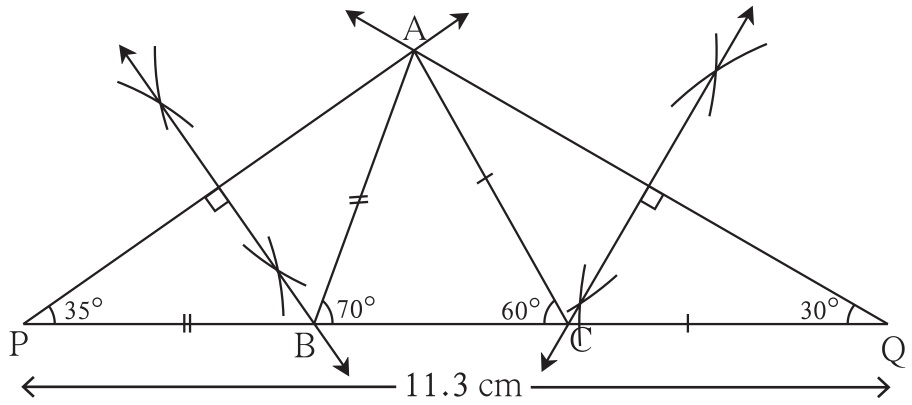
(3) Draw another ray making an angle of 30° at point Q.

(4) Name the point of intersection of the two rays as A.

(5) Draw the perpendicular bisector of seg AP and seg AQ. Name the points as B and C respectively where the perpendicular bisectors intersect line PQ.

(6) Draw seg AB and seg AC.

Δ ABC is the required triangle.



Final Fig. 4.12

Practice set 4.3

1. Construct Δ PQR, in which $\angle Q = 70°$, $\angle R = 80°$ and PQ + QR + PR = 9.5 cm.
2. Construct Δ XYZ, in which $\angle Y = 58°$, $\angle X = 46°$ and perimeter of triangle is 10.5 cm.
3. Construct Δ LMN, in which $\angle M = 60°$, $\angle N = 80°$ and LM + MN + NL = 11 cm.

Problem set 4

1. Construct Δ XYZ, such that XY + XZ = 10.3 cm, YZ = 4.9 cm, $\angle XYZ = 45°$.
2. Construct Δ ABC, in which $\angle B = 70°$, $\angle C = 60°$, AB + BC + AC = 11.2 cm.
3. The perimeter of a triangle is 14.4 cm and the ratio of lengths of its side is 2 : 3 : 4. Construct the triangle.
4. Construct Δ PQR, in which PQ – PR = 2.4 cm, QR = 6.4 cm and $\angle PQR = 55°$.

ICT Tools or Links

Do constructions of above types on the software Geogebra and enjoy the constructions. The third type of construction given above is shown on Geogebra by a different method. Study that method also.

5 Quadrilaterals

Let's study.

- Parallelogram
- Tests of parallelogram
- Rhombus
- Rectangles
- Square
- Trapezium
- Mid point theorem

Let's recall.

1. Write the following pairs considering □ABCD

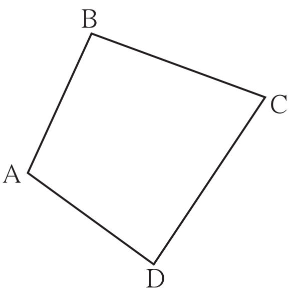


Fig. 5.1

Pairs of adjacent sides:
(1) ... , ... (2) ... , ...
(3) ... , ... (4) ... , ...

Pairs of adjacent angles :
(1) ... , ... (2) ... , ...
(3) ... , ... (4) ... , ...

Pairs of opposite sides (1) ..... , ... (2) ..... , ...
Pairs of opposite angles (1) ..... , ... (2) ..... , ...

Let's recall types of quadrilaterals and their properties .

I am a quadrilateral

My both pairs of opposite sides are parallel

[ ]

My properties

- Opposite sides congruent
- Opposite angels ......
- Diagonals .....

My all angles are right angles

[ ]

My properties

- Opposite sides .....
- Diagonals .....

My all sides are equal in length

[ ]

My properties

- Opposite sides .....
- Diagonals .....

My all angles are equal and all sides are equal

[ ]

My properties

- Diagonals .....

My only one pair of opposite sides is parallel

[ ]

You know different types of quadrilaterals and their properties. You have learned then through different activities like measuring sides and angles, by paper folding method etc. Now we will study these properties by giving their logical proofs.

A property proved logically is called a proof.

In this chapter you will learn that how a rectangle, a rhombus and a square are parallelograms. Let us start our study from parallelogram.

Let's learn.

Parallelogram

A quadrilateral having both pairs of opposite sides parallel is called a parallelogram.

For proving the theorems or for solving the problems we need to draw figure of a parallelogram frequently. Let us see how to draw a parallelogram.

Suppose we have to draw a parallelogram □ABCD.

Method I :

- Let us draw seg AB and seg BC of any length and making an angle of any measure with each other.
- Now we want seg AD and seg BC parallel to each other. So draw a line parallel to seg BC through the point A.
- Similarly we will draw line parallel to AB through the point C. These lines will intersect in point D.

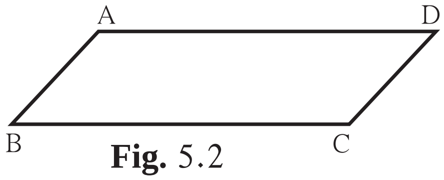


Fig. 5.2

So constructed quadrilateral ABCD will be a parallelogram.

Method II :

- Let us draw seg AB and seg BC of any length and making angle of any measure between them.
- Draw an arc with compass with centre A and radius BC.
- Similarly draw an arc with centre C and radius AB intersecting the arc previously drawn.
- Name the point of intersection of two arcs as D.

Draw seg AD and seg CD.

Quadrilateral so formed is a parallelogram ABCD

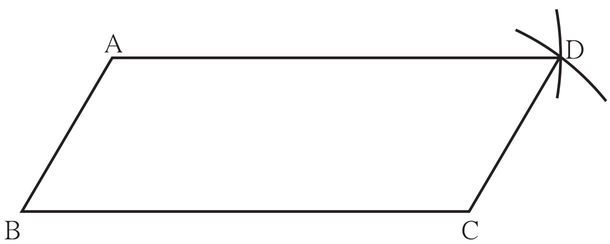


Fig. 5.3

In the second method we have actually drawn □ABCD in which opposite sides are equal. We will prove that a quadrilateral whose opposite sides are equal, is a parallelogram.

Activity I Draw five parallelograms by taking various measures of lengths and angles.

For the proving theorems on parallelogram, we use congruent triangles. To understand how they are used, let's do the following activity.

Activity II

- Draw a parallellogram ABCD on a card sheet. Draw diagonal AC. Write the names of vertices inside the triangle as shown in the figure. Then cut is out.

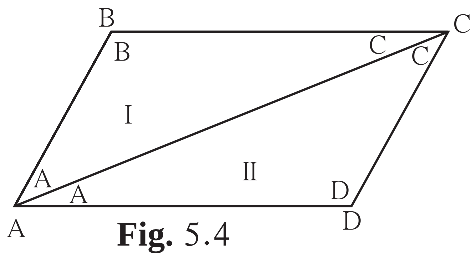


Fig. 5.4

- Fold the quadrilateral on the diagonal AC and see whether ΔADC and ΔCBA match with each other or not.

- Cut □ABCD along diagonals AC and separate ΔADC and ΔCBA. By rotating and flipping ΔCBA, check whether it matchs exactly with ΔADC.

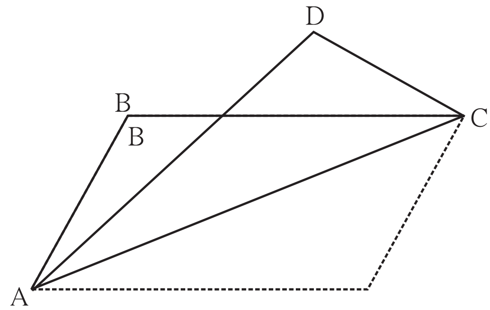


Fig. 5.5

What did you find ? Which sides of ΔCBA match with which sides of ΔADC ? Which angles of ΔCBD match with which angles of ΔADC ?

Side DC matches with side AB and side AD matches with side CB. Similarly ∠B matches with ∠D.

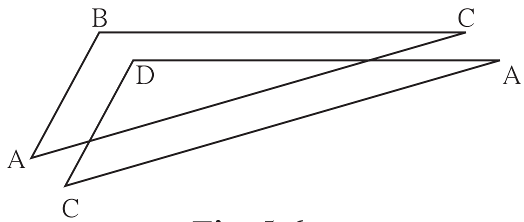


Fig. 5.6

So we can see that opposite sides and angles of a parallelogram are congruent.

We will prove these properties of a parallelogram.

Theorem 1. Opposite sides and opposite angles of a parallelogram are congruent.

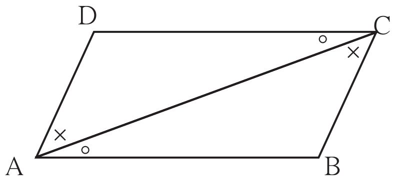


Fig. 5.7

Given : $\square ABCD$ is a parallelogram.
It means side AB $\parallel$ side DC,
side AD $\parallel$ side BC.

To prove : seg AD $\cong$ seg BC ; seg DC $\cong$ seg AB
$\angle ADC \cong \angle CBA$, and $\angle DAB \cong \angle BCD$.

Construction : Draw diagonal AC.

Proof : seg DC $\parallel$ seg AB and diagonal AC is a transversal.

$\therefore$ $\angle DCA \cong \angle BAC$ ....(1)
and $\angle DAC \cong \angle BCA$ ....(2) } ..... alternate angles

Now , in ΔADC and ΔCBA,

$\angle DAC \cong \angle BCA$ .... from (2)

$\angle DCA \cong \angle BAC$ .... from (1)

seg AC $\cong$ seg CA .... common side

$\therefore$ $\Delta ADC \cong \Delta CBA$ .... ASA test

$\therefore$ side AD $\cong$ side CB .... c.s.c.t.

and side DC $\cong$ side AB .... c.s.c.t.,

Also, $\angle ADC \cong \angle CBA$ .... c.a.c.t.

Similarly we can prove $\angle DAB \cong \angle BCD$.

Use your brain power!

In the above theorem, to prove $\angle DAB \cong \angle BCD$, is any change in the construction needed ? If so, how will you write the proof making the change ?

To know one more property of a parallelogram let us do the following activity.

Activity : Draw a parallelogram PQRS. Draw diagonals PR and QS. Denote the intersection of diagonals by letter O. Compare the two parts of each diagonal with a divider. What do you find ?

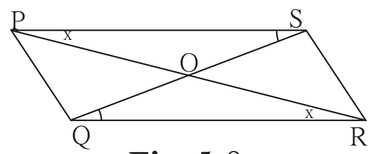


Fig. 5.8

Theorem : Diagonals of a parallelogram bisect each other.

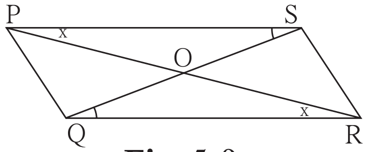


Fig. 5.9

Given : □PQRS is a parallelogram. Diagonals PR and QS intersect in point O.

To prove : seg PO ≅ seg RO,
seg SO ≅ seg QO.

Proof : In ΔPOS and ΔROQ

∠OPS ≅ ∠ORQ .... alternate angles

side PS ≅ side RQ .... opposite sides of parallelogram

∠PSO ≅ ∠RQO .... alternate angles

∴ ΔPOS ≅ ΔROQ .... ASA test

∴ seg PO ≅ seg RO ....
and seg SO ≅ seg QO ....
}..... corresponding sides of congruent triangles

Remember this !

- Adjacent angles of a parallelogram are supplementary.
- Opposite sides of a parallelogram are congruent.
- Opposite angles of a parallelogram are congruent.
- Diagonals of a parallelogram bisect each other.

Solved Examples

Ex (1) □PQRS is a parallelogram. PQ = 3.5, PS = 5.3 ∠Q = 50° then find the lengths of remaining sides and measures of remaining angles.

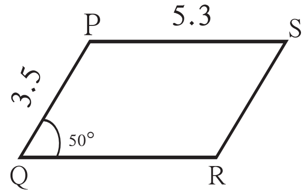


Fig. 5.10

Solution : □PQRS is a parallelogram.

∴ ∠Q + ∠P = 180° .... interior angles are supplementary.

∴ 50° + ∠P = 180°

∴ ∠P = $180^\circ - 50^\circ = 130^\circ$

Now , ∠P = ∠R and ∠Q = ∠S ....opposite angles of a parallelogram.

∴ ∠R = 130° and ∠S = 50°

Similarly, PS = QR and PQ = SR ....opposite sides of a parallelogram.

∴ QR = 5.3 and SR = 3.5

Ex (2) □ABCD is a parallelogram. If $\angle A = (4x + 13)^\circ$ and $\angle D = (5x - 22)^\circ$ then find the measures of $\angle B$ and $\angle C$.

Solution : Adjacent angles of a parallelogram are supplementary.

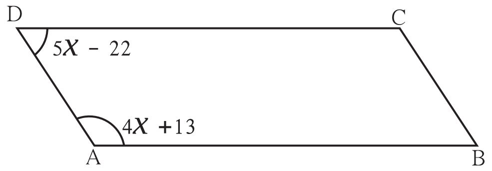


Fig. 5.11

$\angle A$ and $\angle D$ are adjacent angles.

$\therefore (4x + 13)^\circ + (5x - 22)^\circ = 180$

$\therefore 9x - 9 = 180$

$\therefore 9x = 189$

$\therefore x = 21$

$\therefore \angle A = 4x + 13 = 4 \times 21 + 13 = 84 + 13 = 97^\circ$ $\quad \therefore \angle C = 97^\circ$

$\angle D = 5x - 22 = 5 \times 21 - 22 = 105 - 22 = 83^\circ$ $\quad \therefore \angle B = 83^\circ$

Practice set 5.1

1. Diagonals of a parallelogram WXYZ intersect each other at point O. If $\angle XYZ = 135^\circ$ then what is the measure of $\angle XWZ$ and $\angle YZW$?
   If l(OY)= 5 cm then l(WY)= ?

2. In a parallelogram ABCD, If $\angle A = (3x + 12)^\circ$, $\angle B = (2x - 32)^\circ$ then find the value of x and then find the measures of $\angle C$ and $\angle D$.

3. Perimeter of a parallelogram is 150 cm. One of its sides is greater than the other side by 25 cm. Find the lengths of all sides.

4. If the ratio of measures of two adjacent angles of a parallelogram is 1 : 2, find the measures of all angles of the parallelogram.

5*. Diagonals of a parallelogram intersect each other at point O. If AO = 5, BO = 12 and AB = 13 then show that □ABCD is a rhombus.

6. In the figure 5.12, □PQRS and □ABCR are two parallelograms.
   If $\angle P = 110^\circ$ then find the measures of all angles of □ABCR.

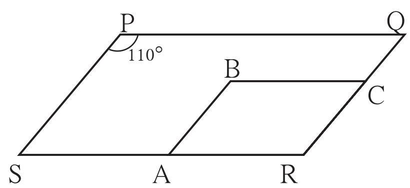


Fig. 5.12

7. In figure 5.13 □ABCD is a parallelogram. Point E is on the ray AB such that BE = AB then prove that line ED bisects seg BC at point F.

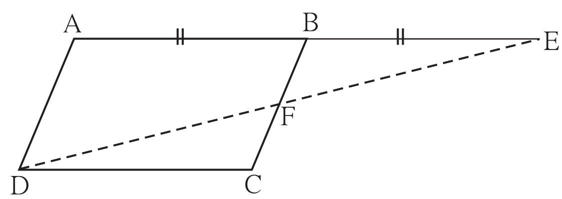


Fig. 5.13

Let's recall.

Tests for parallel lines

1. If a transversal interesects two lines and a pair of corresponding angles is congruent then those lines are parallel.
2. If a transversal intersects two lines and a pair of alternate angles is corgruent then those two lines are parallel.
3. If a transversal intersects two lines and a pair of interior angles is supplementary then those two lines are parallel.

Let's learn.

Tests for parallelogram

Suppose, in □PQRS, PS = QR and PQ = SR and we have to prove that □PQRS is a parallelogram. To prove it, which pairs of sides of □PQRS should be shown parallel ?

Which test can we use to show the sides parallel ? Which line will be convenient as a transversal to obtain the angles necessary to apply the test ?

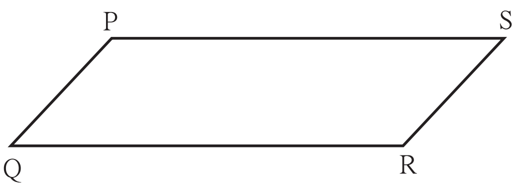


Fig. 5.14

Theorem : If pairs of opposite sides of a quadrilateral are congruent then that quadrilateral is a parallelogram.

Given : In □PQRS

side PS ≅ side QR

side PQ ≅ side SR

To prove : □PQRS is a parallelogram.

Construction : Draw diagonal PR

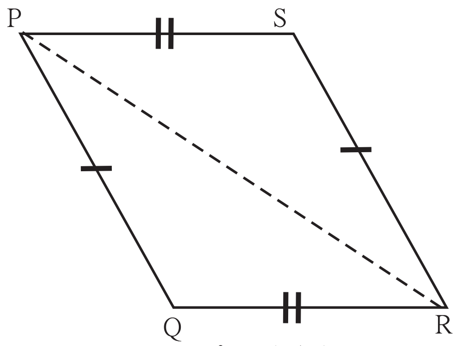


Fig. 5.15

Proof : In Δ SPR and Δ QRP

side PS ≅ side QR ....given

side SR ≅ side QP .... given

side PR ≅ side RP .... common side

∴ Δ SPR ≅ Δ QRP ...... sss test

∴ ∠SPR ≅ ∠QRP .... c.a.c.t.

Similarly, ∠PRS ≅ ∠RPQ ..... c.a.c.t.

∠SPR and ∠QRP are alternate angles formed by the transversal PR of seg PS and seg QR.

$\therefore$ side PS || side QR ......(I) alternate angles test for parallel lines.

Similarly $\angle$PRS and $\angle$RPQ are the alternate angles formed by transversal PR of seg PQ and seg SR.

$\therefore$ side PQ || side SR ......(II) .....alternate angle test

$\therefore$ from (I) and (II) $\square$PQRS is a parallelogram.

On page 56, two methods to draw a parallelogram are given. In the second method actually we have drawn a quadrilateral of which opposite sides are equal. Did you now understand why such a quadrilateral is a parallelogram ?

Theorem : If both the pairs of opposite angles of a quadrilateral are congruent then it is a parallelogram.

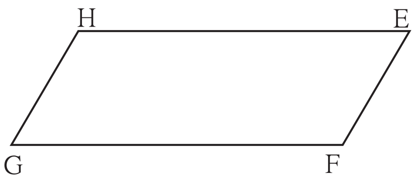


Fig. 5.16

Given : In $\square$EFGH $\angle$E $\cong$ $\angle$G

and $\angle$.......... $\cong$ $\angle$..........

To prove : $\square$EFGH is a ....

Proof : Let $\angle E = \angle G = x$ and $\angle H = \angle F = y$

Sum of all angles of a quadrilateral is ....

$\therefore$ $\angle$E + $\angle$G + $\angle$H + $\angle$F = ...

$\therefore$ $x + y$ + ... + ... = ...

$\therefore$ $\square x + \square y$ = ...

$\therefore$ $x + y = 180^\circ$

$\therefore$ $\angle$G + $\angle$H = ...

$\angle$G and $\angle$H are interior angles formed by transversal HG of seg HE and seg GF.

$\therefore$ side HE || side GF .... (I) interior angle test for parallel lines.

Similarly, $\angle$G + $\angle$F = ...

$\therefore$ side .... || side .... (II) interior angle test for parallel lines.

$\therefore$ From (I) and (II), $\square$EFGH is a ....

Theorem : If the diagonals of a quadrilateral bisect each other then it is a parallelogram.

Given : Diagonals of □ABCD bisect each other in the point E.
It means seg AE ≅ seg CE
and seg BE ≅ seg DE

To prove : □ABCD is a parallelogram.

Proof : Find the answers for the following questions and write the proof of your own.

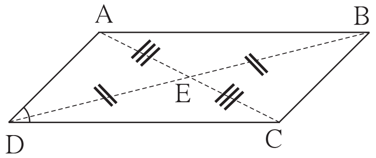


Fig. 5.17

1. Which pair of alternate angles should be shown congruent for proving seg AB || seg DC ? Which transversal will form a pair of alternate angles ?
2. Which triangles will contain the alternate angles formed by the transversal?
3. Which test will enable us to say that the two triangles congruent ?
4. Similarly, can you prove that seg AD || seg BC?

The three theorems above are useful to prove that a given quadrilateral is a parallelogram. Hence they are called as tests of a parallelogram.

One more theorem which is useful as a test for parallelogram is given below.

Theorem : A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.

Given : In □ABCD
seg CB ≅ seg DA and seg CB || seg DA

To prove : □ABCD is a parallelogram.

Construction : Draw diagonal BD.

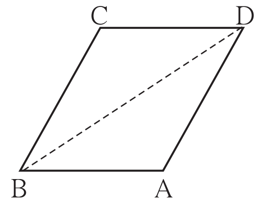


Fig. 5.18

Write the complete proof which is given in short.

Δ CBD ≅ Δ ADB ....SAS test

∴ ∠CDB ≅ ∠ABD ..... c.a.c.t.

∴ seg CD || seg BA ..... alternate angle test for parallel lines

Remember this !

- A quadrilateral is a parallelogram if its pairs of opposite angles are congruent.
- A quadrilateral is a parallelogram if its pairs of opposite sides are congruent.
- A quadrilateral is a parallelogram if its diagonals bisect each other.
- A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and congruent.

These theorems are called tests for parallelogram

Let's recall.

Lines in a note book are parallel. Using these lines how can we draw a parallelogram ?

Solved examples -

Ex (1) □PQRS is parallelogram. M is the midpoint of side PQ and N is the mid point of side RS. Prove that □PMNS and □MQRN are parallelograms.

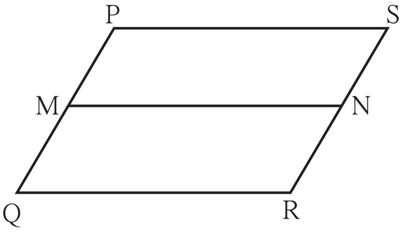


Fig. 5.19

Given : □ PQRS is a parallelogram. M and N are the midpoints of side PQ and side RS respectively.

To prove : □PMNS is a parallelogram.

□MQRN is a parallelogram.

Proof : side PQ || side SR

∴ side PM || side SN ...... (∵ P-M-Q; S-N-R) ......(I)

side PQ ≅ side SR.

∴ $\frac{1}{2}$ side PQ = $\frac{1}{2}$ side SR

∴ side PM ≅ side SN ..... (∵ M and N are midpoints.)......(II)

∴ From (I) and (II), □PMNQ is a parallelogram,

Similarly, we can prove that □MQRN is parallelogram.

Ex (2) Points D and E are the midpoints of side AB and side AC of Δ ABC respectively. Point F is on ray ED such that ED = DF. Prove that □AFBE is a parallelogram. For this example write 'given' and 'to prove' and complete the proof given below.

Given : --------------------

To prove : ----------------------

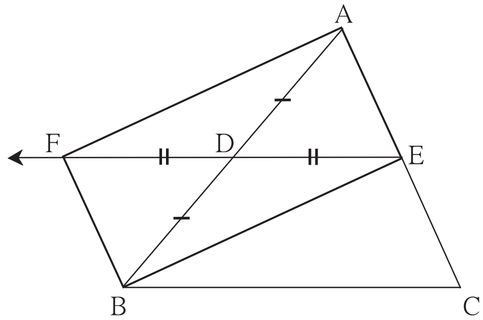


Fig. 5.20

Proof : seg AB and seg EF are ______ of □AFBE.

seg AD ≅ seg DB....... ______

seg ______ ≅ seg ______.......construction.

∴ Diagonals of □AFBE ______ each other

∴ □AFBE is a parallelogram ...by ______ test.

Ex (3) Prove that every rhombus is a parallelogram.

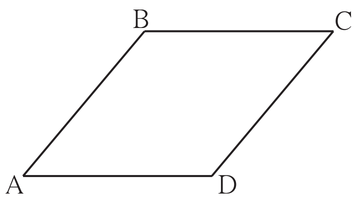


Fig. 5.21

Given : □ABCD is a rhombus

To prove : □ABCD is parallelogram.

Proof : seg AB ≅ seg BC ≅ seg CD ≅ seg DA (given)

∴ side AB ≅ side CD and side BC ≅ side AD

∴ □ABCD is a parallelogram..... opposite side test for parallelogram

Practice set 5.2

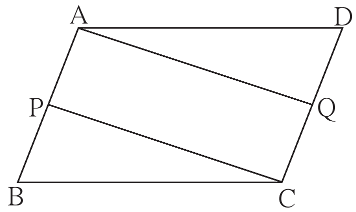


Fig. 5.22

1. In figure 5.22, □ABCD is a parallelogram, P and Q are midpoints of side AB and DC respectively, then prove □APCQ is a parallelogram.

2. Using opposite angles test for parallelogram, prove that every rectangle is a parallelogram.

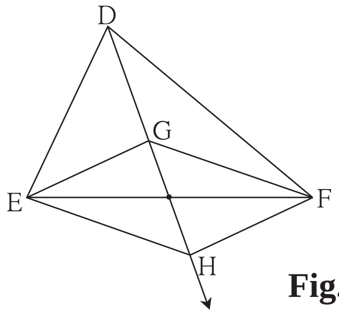


Fig. 5.23

3. In figure 5.23, G is the point of concurrence of medians of Δ DEF. Take point H on ray DG such that D-G-H and DG = GH, then prove that □GEHF is a parallelogram.

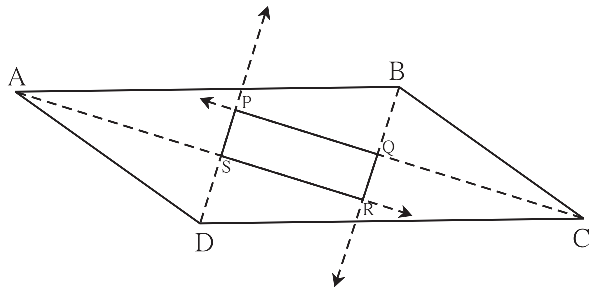


Fig. 5.24

4. Prove that quadrilateral formed by the intersection of angle bisectors of all angles of a parallelogram is a rectangle. (Figure 5.24)

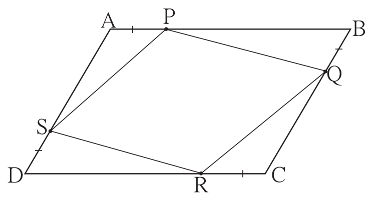


Fig. 5.25

5. In figure 5.25, if points P, Q, R, S are on the sides of parallelogram such that AP = BQ = CR = DS then prove that □PQRS is a parallelogram.

Let's learn.

Properties of rectangle, rhombus and square

Rectangle, rhombus and square are also parallelograms. So the properties that opposite sides are equal, opposite angles are equal and diagonals bisect each other hold good in these types of quadrilaterals also. But there are some more properties of these quadrilaterals.

Proofs of these properties are given in brief. Considering the steps in the given proofs, write the proofs in detail.

Theorem : Diagonals of a rectangle are congruent.

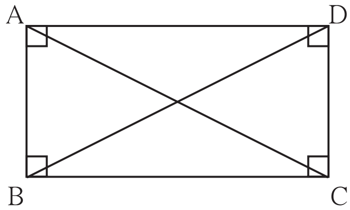


Fig. 5.26

Given : □ABCD is a rectangle

To prove : Diagonal AC ≅ diagonal BD

Proof : Complete the proof by giving suitable reasons.

Δ ADC ≅ Δ DAB ...... SAS test

∴ diagonal AC ≅ diagonal BD..... c.s.c.t.

Theorem : Diagonals of a square are congruent.

Write 'Given', 'To prove' and 'proof' of the theorem.

Theorem : Diagonals of a rhombus are perpendicular bisectors of each other.

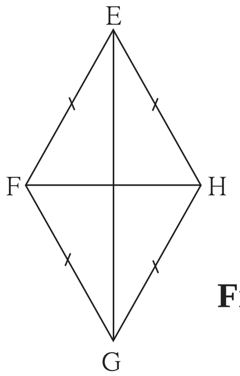


Fig. 5.27

Given : □EFGH is a rhombus

To prove : (i) Diagonal EG is the perpendicular bisector of diagonal HF.

(ii) Diagonal HF is the perpendicular bisector of diagonal EG.

Proof : (i) seg EF ≅ seg EH } given
seg GF ≅ seg GH }

Every point which is equidistant from end points of a segment is on the perpendicular bisector of the segment.

∴ point E and point G are on the perpendicular bisector of seg HF.

One and only one line passes through two distinct points.

∴ line EG is the perpendicular bisector of diagonal HF.

∴ diagonal EG is the perpendicular bisector of diagonal HF.

(ii) Similarly, we can prove that diagonal HF is the perpendicular bisector of EG.

Write the proofs of the following statements.

- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect its opposite angles.
- Diagonals of a square bisect its opposite angles.

Remember this !

- Diagonals of a rectangle are congruent.
- Diagonals of a square are congruent.
- Diagonals of a rhombus are perpendicular bisectors of each other.
- Diagonals of a rhombus bisect the pairs of opposite angles.
- Diagonals of a square are perpendicular bisectors of each other.
- Diagonals of a square bisect opposite angles.

Practice set 5.3

1. Diagonals of a rectangle ABCD intersect at point O. If AC = 8 cm then find the length of BO and if $\angle CAD = 35^\circ$ then find the measure of $\angle ACB$.

2. In a rhombus PQRS if PQ = 7.5 then find the length of QR. If $\angle QPS = 75^\circ$ then find the measure of $\angle PQR$ and $\angle SRQ$.

3. Diagonals of a square IJKL intersects at point M, Find the measures of $\angle IMJ$, $\angle JIK$ and $\angle LJK$.

4. Diagonals of a rhombus are 20 cm and 21 cm respectively, then find the side of rhombus and its perimeter.

5. State with reasons whether the following statements are 'true' or 'false'.

(i) Every parallelogram is a rhombus.

(ii) Every rhombus is a rectangle.

(iii) Every rectangle is a parallelogram.

(iv) Every squre is a rectangle.

(v) Every square is a rhombus.

(vi) Every parallelogram is a rectangle.

Let's learn.

Trapezium

When only one pair of opposite sides of a quadrilateral is parallel then the quadrilateral is called a trapezium.

In the adjacent figure only side AB and side DC of □ABCD are parallel to each other. So this is a trapezium. $\angle A$ and $\angle D$ is a pair of adjacent angles and so is the pair of $\angle B$ and $\angle C$. Therefore by property of parallel lines both the pairs are supplementary.

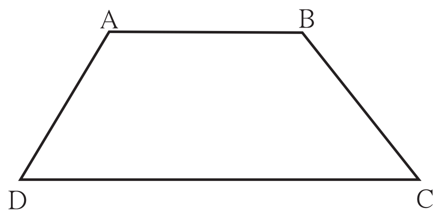


Fig. 5.28

If non-parallel sides of a trapezium are congruent then that quadrilateral is called as an **Isoceles trapezium.**

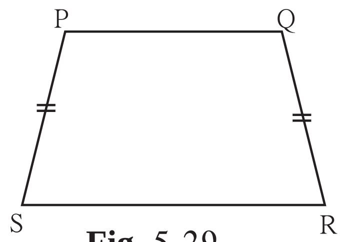


Fig. 5.29

The segment joining the midpoints of non parallel sides of a trapezium is called the median of the trapezium

Solved examples

Ex (1) Measures of angles of □ABCD are in the ratio 4 **:** 5 **:** 7 **:** 8. Show that □ABCD is a trapezium.

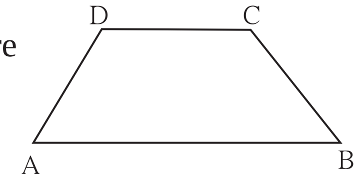


Fig. 5.30

Solution : Let measures of $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are $(4x)^\circ$, $(5x)^\circ$, $(7x)^\circ$, and $(8x)^\circ$ respectively.

Sum of all angles of a quadrialteral is 360°.

$\therefore\ 4x + 5x + 7x + 8x = 360$

$\therefore\ 24x = 360 \quad \therefore\ x = 15$

$\angle A = 4 \times 15 = 60^\circ$, $\angle B = 5 \times 15 = 75^\circ$, $\angle C = 7 \times 15 = 105^\circ$,

and $\angle D = 8 \times 15 = 120^\circ$

Now , $\angle B + \angle C = 75^\circ + 105^\circ = 180^\circ$

$\therefore$ side CD $\parallel$ side BA...... (I)

But $\angle B + \angle A = 75^\circ + 60^\circ = 135^\circ \neq 180^\circ$

$\therefore$ side BC and side AD are not parallel ....(II)

$\therefore$ □ABCD is a trapezium. ....[from (I) and (II)]

Ex (2) In □PQRS, side PS $\parallel$ side QR and side PQ $\cong$ side SR, side QR > side PS then prove that $\angle PQR \cong \angle SRQ$

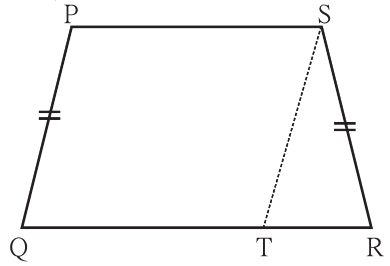


Fig. 5.31

Given : In □PQRS, side PS $\parallel$ side QR, side PQ $\cong$ side SR and side QR > side PS.

To prove : $\angle PQR \cong \angle SRQ$

Construction : Draw the segment parallel to side PQ through the point S which intersects side QR in T.

Proof : In □PQRS,

seg PS $\parallel$ seg QT ....given

seg PQ $\parallel$ seg ST ....construction

$\therefore$ □PQTS is a parallelogram

$\therefore \angle PQT \cong \angle STR$ ..... corresponding angles (I)

and seg PQ $\cong$ seg ST ......opposite sides of parallelogram

But seg PQ $\cong$ seg SR ......given

$\therefore$ seg ST $\cong$ seg SR

$\therefore \angle STR \cong \angle SRT$.....isosceles triangle theorem (II)

$\therefore \angle PQT \cong \angle SRT$ ....[from (I) and (II)]

$\therefore\ \angle PQR \cong \angle SRQ$

Hence, it is proved that base angles of an isosceles trapezium are congruent.

Practice set 5.4

1. In □IJKL, side IJ || side KL $\angle I = 108^\circ$ $\angle K = 53^\circ$ then find the measures of $\angle J$ and $\angle L$.
2. In □ABCD, side BC || side AD, side AB ≅ side DC If $\angle A = 72^\circ$ then find the measures of $\angle B$, and $\angle D$.
3. In □ABCD, side BC < side AD (Figure 5.32) side BC || side AD and if side BA ≅ side CD then prove that ∠ABC ≅ ∠DCB.

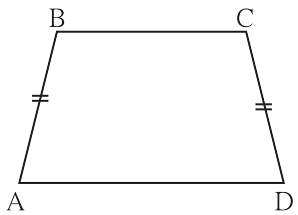


Fig. 5.32

Let's learn.

Theorem of midpoints of two sides of a triangle

Statement : The segment joining midpoints of any two sides of a triangle is parallel to the third side and half of it.

Given : In Δ ABC, point P is the midpoint of seg AB and point Q is the midpoint of seg AC

To prove : seg PQ || seg BC

and $PQ = \frac{1}{2} BC$

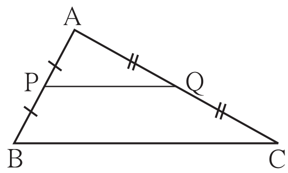


Fig. 5.33

Construction : Produce seg PQ upto R such that PQ = QR

Draw seg RC.

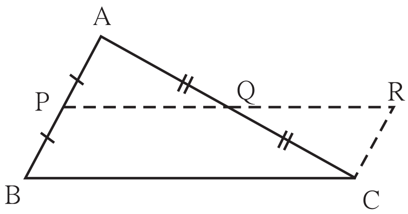


Fig. 5.34

Proof : In Δ AQP and Δ CQR

seg PQ ≅ seg QR ...... construction

seg AQ ≅ seg QC ...... given

∠AQP ≅ ∠CQR ..... vertically opposite angles.

∴ Δ AQP ≅ Δ CQR .... SAS test

∠PAQ ≅ ∠RCQ ..... (1) c.a.c.t.

∴ seg AP ≅ seg CR ......(2) c.s.c.t.

From (1) line AB || line CR.........alternate angle test

from (2) seg AP ≅ seg CR

Now, seg AP ≅ seg PB ≅ seg CR and seg PB || seg CR

∴ □PBCR is a parallelogram.

∴ seg PQ || seg BC and PR = BC ..... opposite sides are congruent

$PQ = \frac{1}{2} PR$ ...... (construction)

$\therefore\ PQ = \frac{1}{2} BC \qquad \because\ PR = BC$

Converse of midpoint theroem

Theorem : If a line drawn through the midpoint of one side of a triangle and parallel to the other side then it bisects the third side.

For this theorem 'Given', To prove', 'construction' is given below. Try to write the proof.

Given : Point D is the midpoint of side AB of Δ ABC. Line l passing through the point D and parallel to side BC intersects side AC in point E.

To prove : AE = EC

Construction : Take point F on line l such that D-E-F and DE = EF. Draw seg CF.

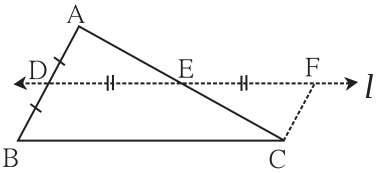


Fig. 5.35

Proof : Use the construction and line $l \parallel$ seg BC which is given. Prove Δ ADE $\cong$ Δ CFE and complete the proof.

Ex (1) Points E and F are mid points of seg AB and seg AC of Δ ABC respectively. If EF = 5.6 then find the length of BC.

Solution : In Δ ABC, point E and F are midpoints of side AB and side AC respectively.

$EF = \frac{1}{2} BC$ ....midpoint theorem

$5.6 = \frac{1}{2} BC \qquad \therefore\ BC = 5.6 \times 2 = 11.2$

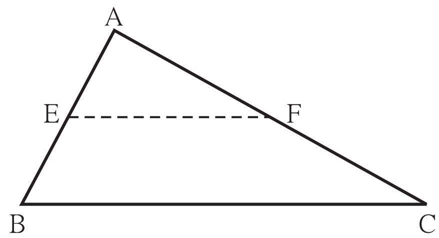


Fig. 5.36

Ex (2) Prove that the quadrilateral formed by joining the midpoints of sides of a quadrilateral in order is a parallelogram.

Given : □ABCD is a quadrilateral. P, Q, R, S are midpoints of the sides AB, BC, CD and AD respectively.

To prove : □PQRS is a parallelogram.

Construction : Draw diagonal BD

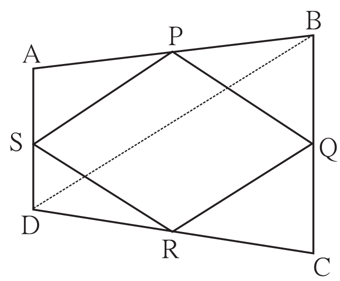


Fig. 5.37

Proof : In Δ ABD, the midpoint of side AD is S and the midpoint of side AB is P.

$\therefore$ by midpoint theorem, PS || DB and PS = $\frac{1}{2}$ BD .... (1)

In Δ DBC point Q and R are midpoints of side BC and side DC respectively.

$\therefore$ QR || BD and QR = $\frac{1}{2}$ BD ....by midpoint theorem (2)

$\therefore$ PS || QR and PS = QR .... from (1) and (2)

$\therefore$ □PQRS is a parallelogram.

Practice set 5.5

1. In figure 5.38, points X, Y, Z are the midpoints of side AB, side BC and side AC of Δ ABC respectively. AB = 5 cm, AC = 9 cm and BC = 11 cm. Find the length of XY, YZ, XZ.

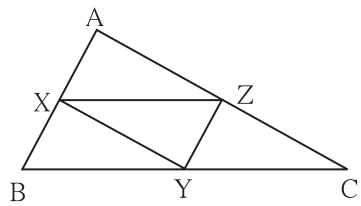


Fig. 5.38

2. In figure 5.39, □PQRS and □MNRL are rectangles. If point M is the midpoint of side PR then prove that, (i) SL = LR, (ii) LN = $\frac{1}{2}$ SQ.

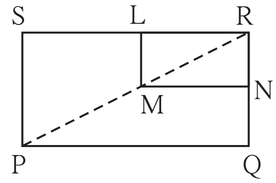


Fig. 5.39

3. In figure 5.40, ΔABC is an equilateral traingle. Points F,D and E are midpoints of side AB, side BC, side AC respectively. Show that Δ FED is an equilateral traingle.

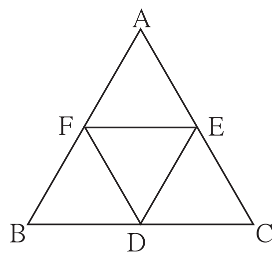


Fig. 5.40

4. In figure 5.41, seg PD is a median of Δ PQR. Point T is the mid point of seg PD. Produced QT intersects PR at M. Show that $\frac{PM}{PR} = \frac{1}{3}$.
[Hint : draw DN || QM.]

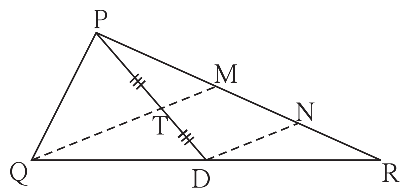


Fig. 5.41

Problem set 5

1. Choose the correct alternative answer and fill in the blanks.

(i) If all pairs of adjacent sides of a quadrilateral are congruent then it is called ....
(A) rectangle (B) parallelogram (C) trapezium, (D) rhombus

(ii) If the diagonal of a square is $12\sqrt{2}$ cm then the perimeter of square is ......

(A) 24 cm (B) $24\sqrt{2}$ cm (C) 48 cm (D) $48\sqrt{2}$ cm

(iii) If opposite angles of a rhombus are $(2x)^\circ$ and $(3x - 40)^\circ$ then value of x is ...

(A) 100 ° (B) 80 ° (C) 160 ° (D) 40 °

2. Adjacent sides of a rectangle are 7 cm and 24 cm. Find the length of its diagonal.

3. If diagonal of a square is 13 cm then find its side.

4. Ratio of two adjacent sides of a parallelogram is 3 : 4, and its perimeter is 112 cm. Find the length of its each side.

5. Diagonals PR and QS of a rhombus PQRS are 20 cm and 48 cm respectively. Find the length of side PQ.

6. Diagonals of a rectangle PQRS are intersecting in point M. If $\angle$QMR = 50° then find the measure of $\angle$MPS.

7. In the adjacent Figure 5.42, if seg AB $\parallel$ seg PQ , seg AB $\cong$ seg PQ, seg AC $\parallel$ seg PR, seg AC $\cong$ seg PR then prove that, seg BC $\parallel$ seg QR and seg BC $\cong$ seg QR.

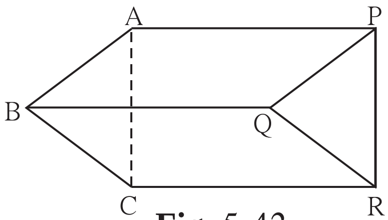


Fig. 5.42

8*. In the Figure 5.43, □ABCD is a trapezium. AB $\parallel$ DC. Points P and Q are midpoints of seg AD and seg BC respectively. Then prove that, PQ $\parallel$ AB and

$$PQ = \frac{1}{2}(AB + DC).$$

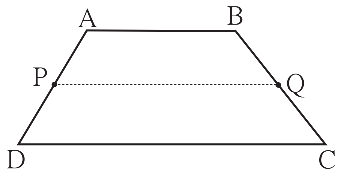


Fig. 5.43

9. In the adjacent figure 5.44, □ABCD is a trapezium. AB $\parallel$ DC. Points M and N are midpoints of diagonal AC and DB respectively then prove that MN $\parallel$ AB.

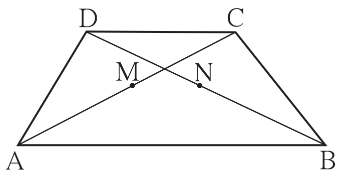


Fig. 5.44

Activity

To verify the different properties of quadrilaterals

Material : A piece of plywood measuring about 15 cm× 10 cm, 15 thin screws, twine, scissor.

Note : On the plywood sheet, fix five screws in a horizontal row keeping a distance of 2cm between any two adjacent screws. Similarly make two more rows of screws exactly below the first one. Take care that the vertical distance between any two adjacent screws is also 2cm.

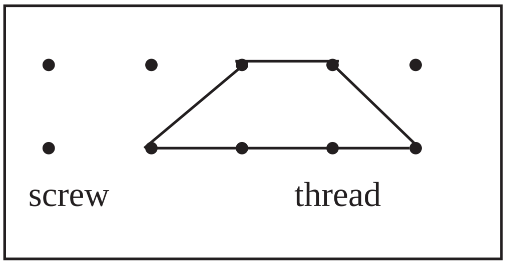


Fig. 5.45

With the help of the screws, make different types of quadrilaterals of twine. Verify the properties of sides and angles of the quadrilaterals.

Additional information

You know the property that the point of concurrence of medians of a triangle divides the medians in the ratio 2 : 1. Proof of this property is given below.

Given : seg AD and seg BE are the medians of Δ ABC which intersect at point G.

To prove: AG : GD = 2 : 1

Construction : Take point F on ray AD such that G-D-F and GD = DF

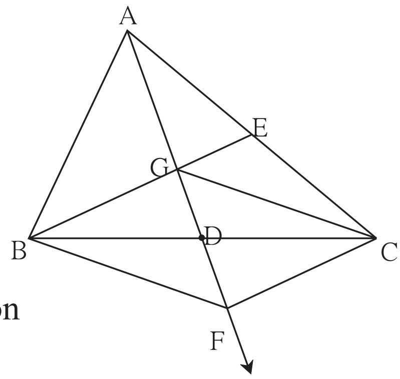


Fig. 5.46

Proof : Diagonals of □BGCF bisect each other .... given and construction

∴ □BGCF is a parallelogram.

∴ seg BE || set FC

Now point E is the midpoint of side AC of Δ AFC . ... given

seg EB || seg FC

Line passing through midpoint of one side and parallel to the other side bisects the third side.

∴ point G is the midpoint of side AF.

∴ AG = GF

But GF = 2GD .... construction

∴ AG = 2 GD

∴ $\frac{AG}{GD} = \frac{2}{1}$ i.e. AG : GD = 2 : 1

6 Circle

24EZND

Let's study.

- Circle
- Property of chord of the circle
- Incircle
- Circumcircle

Let's recall.

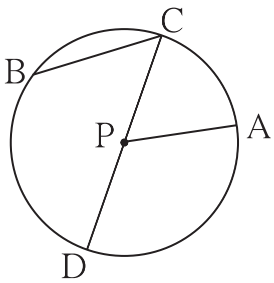


Fig. 6.1

In adjoining figure, observe the circle with center P. With reference to this figure, complete the following table.

---	seg PA	---	---	---	---	∠CPA
chord	---	diameter	radius	centre	central angle	---

Let's learn.

Circle

Let us describe this circle in terms of a set of points.

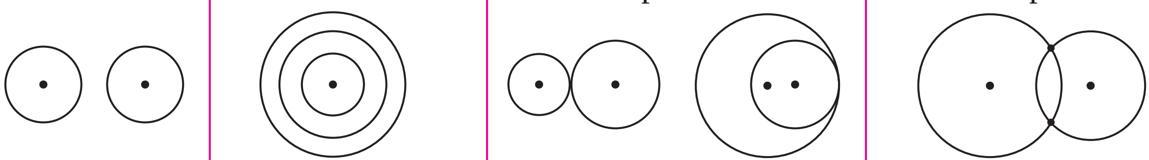
- The set of points in a plane which are equidistant from a fixed point in the plane is called a circle.

Some terms related with a circle.

- The fixed point is called the centre of the circle.
- The segment joining the centre of the circle and a point on the circle is called a radius of the circle.
- The distance of a point on the circle from the centre of the circle is also called the radius of the circle.
- The segment joining any two points of the circle is called a chord of the circle.
- A chord passing through the centre of a circle is called a diameter of the circle.
  A diameter is a largest chord of the circle.

Circles in a plane

Congruent circles	Concentric circles	Circles intersecting in a point	Circles intersecting in two points



• the same radii	• the same centre, different radii	• different centres, different radii, only one common point	• different centres, different radii, two common points

Fig. 6.2

Let's learn.

Properties of chord

Activity I : Every student in the group will do this activity. Draw a circle in your notebook. Draw any chord of that circle. Draw perpendicular to the chord through the centre of the circle. Measure the lengths of the two parts of the chord. Group leader will prepare a table and other students will write their observations in it.

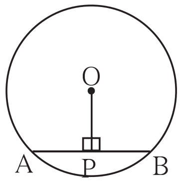


Fig. 6.3

Student / Length	1	2	3	4	5	6
l (AP)	...... cm					
l (PB)	...... cm					

Write theproperty which you have observed.
Let us write the proof of this property.

Theorem : A perpendicular drawn from the centre of a circle on its chord bisects the chord.

Given : seg AB is a chord of a circle with centre O.
seg OP $\perp$ chord AB

To prove : seg AP $\cong$ seg BP

Proof : Draw seg OA and seg OB

In Δ OPA and Δ OPB

$\angle$OPA $\cong$ $\angle$OPB . . . . seg OP $\perp$ chord AB

seg OP $\cong$ seg OP . . . . common side

hypotenuse OA $\cong$ hypotenuse OB . . . . radii of the same circle

$\therefore$ Δ OPA $\cong$ Δ OPB . . . . hypotenuse side theorem

seg PA $\cong$ seg PB . . . . c.s.c.t.

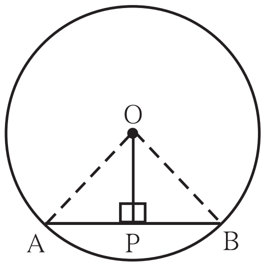


Fig. 6.4

Activity II : Every student from the group will do this activity. Draw a circle in your notebook. Draw a chord of the circle. Join the midpoint of the chord and centre of the circle. Measure the angles made by the segment with the chord. Discuss about the measures of the angles with your friends.
Which property do the observations suggest ?

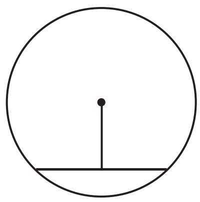


Fig. 6.5

Theorem : The segment joining the centre of a circle and the midpoint of its chord is perpendicular to the chord.

Given : seg AB is a chord of a circle with centre O and P is the midpoint of chord AB of the circle. That means seg AP ≅ seg PB.

To prove : seg OP ⊥ chord AB

Proof : Draw seg OA and seg OB.

In Δ AOP and Δ BOP

seg OA ≅ seg OB . . . . radii of the same circle

seg OP ≅ seg OP. . . . . common sides

seg AP ≅ seg BP . . . . given

∴ Δ AOP ≅ Δ BOP . . . . SSS test

∴ ∠OPA ≅ ∠OPB . . . . c.a.c.t. . . .(I)

Now ∠OPA + ∠OPB = 180° . . . angles in linear pair

∴ ∠OPB + ∠OPB = 180° . . . . . . from (I)

∴ 2 ∠OPB = 180°

∴ ∠OPB = 90°

∴ seg OP ⊥ chord AB

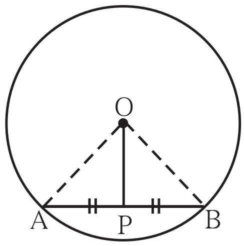


Fig. 6.6

Solved examples

Ex (1) Radius of a circle is 5 cm. The length of a chord of the circle is 8 cm. Find the distance of the chord from the centre.

Solution :

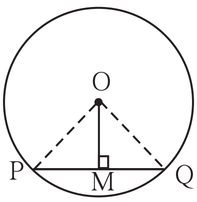


Fig. 6.7

Let us draw a figure from the given information.
O is the centre of the circle.
Length of the chord is 8 cm.
seg OM ⊥ chord PQ.

We know that a perpendicular drawn from the centre of a circle on its chord bisects the chord.

∴ PM = MQ = 4 cm

Radius of the circle is 5 cm, means OQ = 5 cm .... given

In the right angled Δ OMQ using Pythagoras' theorem,

$OM^2 + MQ^2 = OQ^2$

∴ $OM^2 + 4^2 = 5^2$

∴ $OM^2 = 5^2 - 4^2 = 25 - 16 = 9 = 3^2$

∴ OM = 3

Hence distance of the chord from the centre of the circle is 3 cm.

Ex (2) Radius of a circle is 20 cm. Distance of a chord from the centre of the circle is 12 cm. Find the length of the chord.

Solution : Let the centre of the circle be O. Radius = OD = 20 cm.

Distance of the chord CD from O is12 cm. seg OP $\perp$ seg CD

$\therefore$ OP = 12 cm

Now CP = PD ...... perpendicular drawn from the centre bisects the chord

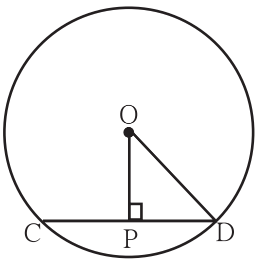


Fig. 6.8

In the right angled Δ OPD, using Pythagoras' theorem

$OP^2 + PD^2 = OD^2$

$(12)^2 + PD^2 = 20^2$

$PD^2 = 20^2 - 12^2$

$PD^2 = (20+12)(20-12)$

$= 32 \times 8 = 256$

$\therefore$ PD = 16 $\qquad$ $\therefore$ CP = 16

CD = CP + PD = 16 + 16 = 32

$\therefore$ the length of the chord is 32 cm.

Practice set 6.1

1. Distance of chord AB from the centre of a circle is 8 cm. Length of the chord AB is 12 cm. Find the diameter of the circle.
2. Diameter of a circle is 26 cm and length of a chord of the circle is 24 cm. Find the distance of the chord from the centre.
3. Radius of a circle is 34 cm and the distance of the chord from the centre is 30 cm, find the length of the chord.
4. Radius of a circle with centre O is 41 units. Length of a chord PQ is 80 units, find the distance of the chord from the centre of the circle.
5. In figure 6.9, centre of two circles is O. Chord AB of bigger circle intersects the smaller circle in points P and Q. Show that AP = BQ
6. Prove that, if a diameter of a circle bisects two chords of the circle then those two chords are parallel to each other.

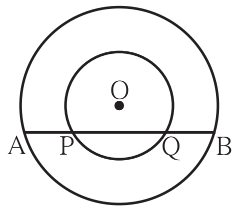


Fig. 6.9

Activity I

(1) Draw circles of convenient radii.
(2) Draw two equal chords in each circle.
(3) Draw perpendicular to each chord from the centre.
(4) Measure the distance of each chord from the centre.

Let's learn.

Relation between congruent chords of a circle and their distances from the centre

Activity II : Measure the lengths of the perpendiculars on chords in the following figures.

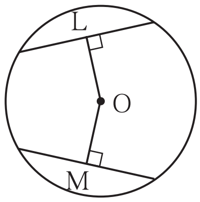


Figure (i)

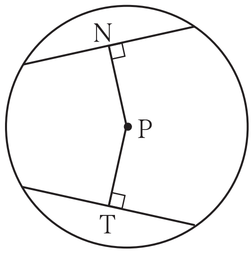


Figure (ii)

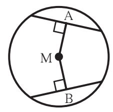


Figure (iii)

Did you find OL = OM in fig (i), PN = PT in fig (ii) and MA = MB in fig (iii) ?

Write the property which you have noticed from this activity.

Let's learn.

Properties of congruent chords

Theorem : Congruent chords of a circle are equidistant from the centre of the circle.

Given : In a circle with centre O

chord AB ≅ chord CD

OP ⊥ AB, OQ ⊥ CD

To prove : OP = OQ

Construction : Join seg OA and seg OD.

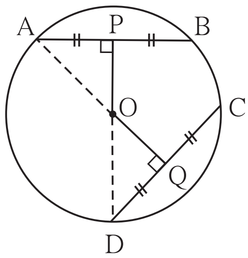


Fig. 6.10

Proof : AP = $\frac{1}{2}$ AB, DQ = $\frac{1}{2}$ CD . . . perpendicular drawn from the centre of a circle to its chord bisects the chord.

AB = CD . . . . given

∴ AP = DQ

∴ seg AP ≅ seg DQ . . . . (I) . . . segments of equal lengths

In right angled Δ APO and right angled Δ DQO

seg AP ≅ seg DQ . . . . from (I)

hypotenuse OA ≅ hypotenuse OD . . . . radii of the same circle

∴ Δ APO ≅ Δ DQO . . . . hypotenuse side theorem

seg OP ≅ seg OQ . . . . c.s.c.t.

∴ OP = OQ . . . . Length of congruent segments.

Congruent chords in a circle are equidistant from the centre of the circle.

Theorem : The chords of a circle equidistant from the centre of a circle are congruent.

Given : In a circle with centre O

seg OP $\perp$ chord AB

seg OQ $\perp$ chord CD

and OP = OQ

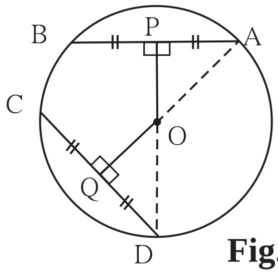


Fig. 6.11

To prove : chord AB $\cong$ chord CD

Construction : Draw seg OA and seg OD.

Proof : (Complete the proof by filling in the gaps.)

In right angled Δ OPA and right Δ OQD

hypotenuse OA $\cong$ hypotenuse OD . . . . [ ]

seg OP $\cong$ seg OQ . . . . given

$\therefore$ Δ OPA $\cong$ Δ OQD . . . . [ ]

$\therefore$ seg AP $\cong$ seg QD . . . . c.s.c.t.

$\therefore$ AP = QD . . . . (I)

But AP = $\frac{1}{2}$ AB, and DQ = $\frac{1}{2}$ CD . . . . . [ ]

and AP = QD . . . . from (I)

$\therefore$ AB = CD

$\therefore$ seg AB $\cong$ seg CD

Note that both the theorems are converses of each other

Remember this !

Congruent chords of a circle are equidistant from the centre of the circle.
The chords equidistant from the centre of a circle are congruent.

Activity : The above two theorems can be proved for two congruent circles also.

1. Congruent chords in congruent circles are equidistant from their respective centres.
2. Chords of congruent circles which are equidistant from their respective centres are congruent.
   Write 'Given', 'To prove' and the proofs of these theorems .

Solved example

Ex. In the figure 6.12, O is the centre of the circle and AB = CD. If OP = 4 cm, find the length of OQ.

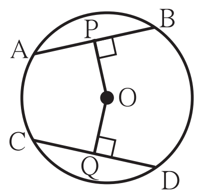


Fig. 6.12

Solution : O is the centre of the circle,

chord AB $\cong$ chord CD .....given

OP $\perp$ AB, OQ $\perp$ CD

OP = 4 cm, means distance of AB from the centre O is 4 cm.

The congruent chords of a circle are equidistant from the centre of the circle.

$\therefore$ OQ = 4 cm

Practice set 6.2

1. Radius of circle is 10 cm. There are two chords of length 16 cm each. What will be the distance of these chords from the centre of the circle ?
2. In a circle with radius 13 cm, two equal chords are at a distance of 5 cm from the centre. Find the lengths of the chords.
3. Seg PM and seg PN are congruent chords of a circle with centre C. Show that the ray PC is the bisector of $\angle$NPM.

Let's recall.

In previous standard we have verified the property that the angle bisectors of a triangle are concurrent. We denote the point of concurrence by letter I.

Let's learn.

Incircle of a triangle

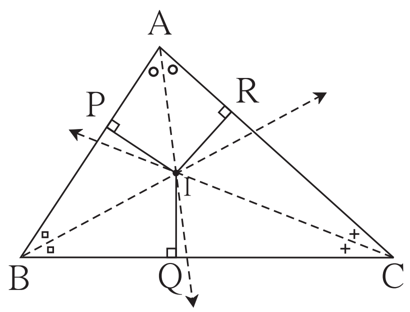


Fig. 6.13

In fig. 6.13, bisectors of all angles of a ΔABC meet in the point I. Perpendiculars on three sides are drawn from the point of concurrence.

$IP \perp AB, \quad IQ \perp BC, \quad IR \perp AC$

We know that, every point on the angle bisector is equidistant from the sides of the angle.

Point I is on the bisector of $\angle$B. $\therefore$ IP = IQ.

Point I is on the bisector of $\angle$C $\therefore$ IQ = IR

$\therefore$ IP = IQ= IR

That is point I is equidistant from all the sides of ΔABC.

$\therefore$ if we draw a circle with centre I and radius IP, it will touch the sides AB, AC, BC of ΔABC internally.

This circle is called the Incircle of the triangle ABC.

Let's learn.

To construct the incircle of a triangle

Ex. Construct Δ PQR such that PQ = 6 cm, $\angle Q = 35^\circ$, QR = 5.5 cm. Draw incircle of Δ PQR.

Draw a rough figure and show all measures in it.

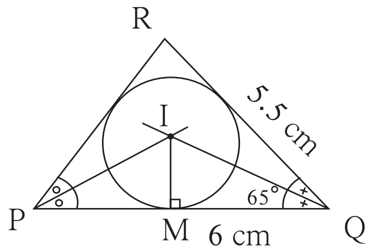
(1) Construct Δ PQR of given measures.

(2) Draw bisectors of any two angles of the triangle.

(3) Denote the point of intersection of angle bisectors as I.

(4) Draw perpendicular IM from the point I to the side PQ.

(5) Draw a circle with centre I and radius IM.



Rough fig. 6.14

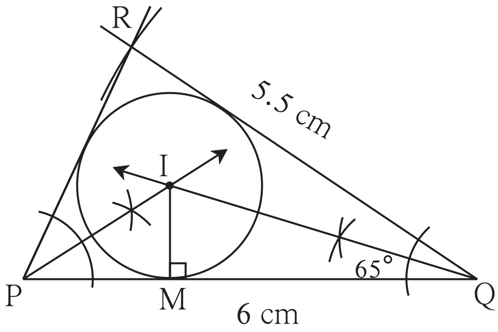


Fig. 6.15

Remember this !

The circle which touches all the sides of a triangle is called incircle of the triangle and the centre of the circle is called the incentre of the triangle.

Let's recall.

In previous standards we have verified the property that perpendicular bisectors of sides of a triangle are concurrent. That point of concurrence is denoted by the letter C.

Let's learn.

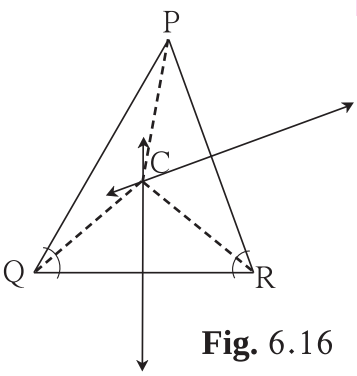


Fig. 6.16

In fig. 6.16, the perpendicular bisectors of sides of Δ PQR are intersecting at point C. So C is the point of concurrence of perpendicular bisectors.

Circumcircle

Point C is on the perpendicular bisectors of the sides of triangle PQR. Join PC, QC and RC. We know that, every point on the perpendicular bisector is equidistant from the end points of the segment.

Point C is on the perpendicular bisector of seg PQ. $\therefore$ PC = QC I

Point C is on the perpendicular bisector of seg QR. $\therefore$ QC = RC II

$\therefore$ PC = QC = RC . . . . . From I and II

$\therefore$ the circle with centre C and radius PC will pass through all the vertices of Δ PQR. This circle is called as the circumcircle.

Remember this !

Circle passing through all the vertices of a triangle is called circumcircle of the triangle and the centre of the circle is called the circumcentre of the triangle.

Let's learn.

To draw the circumcircle of a triangle

Ex. Construct Δ DEF such that DE = 4.2 cm, $\angle D = 60^\circ$, $\angle E = 70^\circ$ and draw circumcircle of it. Draw rough figure. Write the given measures.

Rough figure

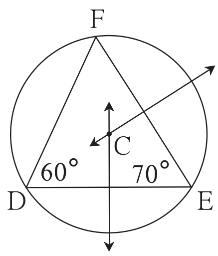


Fig. 6.17

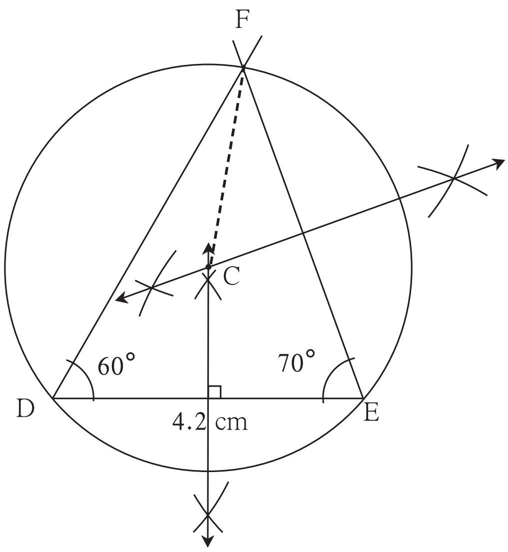


Fig. 6.18

Steps of construction **:**

(1) Draw Δ DEF of given measures.

(2) Draw perpendicular bisectors of any two sides of the triangle.

(3) Name the point of intersection of perpendicular bisectors as C.

(4) Join seg CF.

(5) Draw circle with centre C and radius CF.

Activity :
Draw different triangles of different measures and draw incircles and circumcircles of them. Complete the table of observations and discuss.

Type of triangle	Equilateral triangle	Isosceles triangles	Scalene triangle
Position of incenter	Inside the triangle	Inside the triangle	Inside the triangle
Position of circumcentre	Inside the triangle	Inside, outside on the triangle	

Type of triangle	Acute angled triangle	Right angled triangle	Obtuse angled triangle
Position of incentre			
Position of circumcircle		Midpoint of hypotenuse	

Remember this !

- Incircle of a triangle touches all sides of the triangle from inside.
- For construction of incircle of a triangle we have to draw bisectors of any two angles of the triangle.
- Circumcircle of a triangle passes through all the vertices of a triangle.
- For construciton of a circumcircle of a triangle we have to draw perpendicular bisectors of any two sides of the triangle.
- Circumcentre of an acute angled triangle lies inside the triangle.
- Circumcentre of a right angled triangle is the midpoint of its hypotenuse.
- Circumcentre of an obtuse angled triangle lies in the exterior of the triangle.
- Incentre of any triangle lies in the interior of the triangle.

Activity : Draw any equilateral triangle. Draw incircle and circumcircle of it. What did you observe while doing this activity ?

(1) While drawing incircle and circumcircle, do the angle bisectors and perpendicular bisectors coincide with each other ?

(2) Do the incentre and circumcenter coincide with each other ? If so, what can be the reason of it ?

(3) Measure the radii of incircle and circumcircle and write their ratio.

Remember this !

- The perpendicular bisectors and angle bisectors of an equilateral triangle are coincedent.
- The incentre and the circumcentre of an equilateral triangle are coincedent.
- Ratio of radius of circumcircle to the radius of incircle of an equilateral triangle is 2 : 1

Practice set 6.3

1. Construct Δ ABC such that $\angle$B =100°, BC = 6.4 cm, $\angle$C = 50° and construct its incircle.
2. Construct Δ PQR such that $\angle$P = 70°, $\angle$R = 50°, QR = 7.3 cm. and construct its circumcircle.
3. Construct Δ XYZ such that XY = 6.7 cm, YZ = 5.8 cm, XZ = 6.9 cm. Construct its incircle.
4. In Δ LMN, LM = 7.2 cm, $\angle$M = 105°, MN = 6.4 cm, then draw Δ LMN and construct its circumcircle.
5. Construct Δ DEF such that DE = EF = 6 cm, $\angle$F = 45° and construct its circumcircle.

◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇ Problem set 6 ◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇◇

1. Choose correct alternative answer and fill in the blanks.

(i) Radius of a circle is 10 cm and distance of a chord from the centre is 6 cm. Hence the length of the chord is ....

(A) 16 cm (B) 8 cm (C) 12 cm (D) 32 cm

(ii) The point of concurrence of all angle bisectors of a triangle is called the ......

(A) centroid (B) circumcentre (C) incentre (D) orthocentre

(iii) The circle which passes through all the vertices of a triangle is called .....

(A) circumcircle (B) incircle (C) congruent circle (D) concentric circle

(iv) Length of a chord of a circle is 24 cm. If distance of the chord from the centre is 5 cm, then the radius of that circle is ....

(A) 12 cm (B) 13 cm (C) 14 cm (D) 15 cm

(v) The length of the longest chord of the circle with radius 2.9 cm is .....

(A) 3.5 cm (B) 7 cm (C) 10 cm (D) 5.8 cm

(vi) Radius of a circle with centre O is 4 cm. If l(OP) = 4.2 cm, say where point P will lie.

(A) on the centre (B) Inside the circle (C) outside the circle(D) on the circle

(vii) The lengths of parallel chords which are on opposite sides of the centre of a circle are 6 cm and 8 cm. If radius of the circle is 5 cm, then the distance between these chords is .....

(A) 2 cm (B) 1 cm (C) 8 cm (D) 7 cm

2. Construct incircle and circumcircle of an equilateral Δ DSP with side 7.5 cm. Measure the radii of both the circles and find the ratio of radius of circumcircle to the radius of incircle.

3. Construct Δ NTS where NT = 5.7 cm, TS = 7.5 cm and $\angle$NTS = 110° and draw incircle and circumcircle of it.

4. In the figure 6.19, C is the centre of the circle. seg QT is a diameter CT = 13, CP = 5, find the length of chord RS.

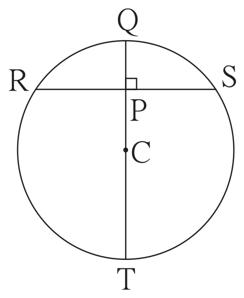


Fig. 6.19

5. In the figure 6.20, P is the centre of the circle. chord AB and chord CD intersect on the diameter at the point E If $\angle$AEP $\cong$ $\angle$DEP then prove that AB = CD.

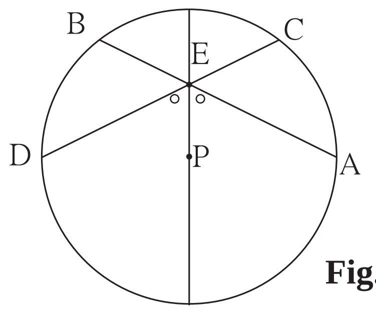


Fig. 6.20

6. In the figure 6.21, CD is a diameter of the circle with centre O. Diameter CD is perpendicular to chord AB at point E. Show that Δ ABC is an isosceles triangle.

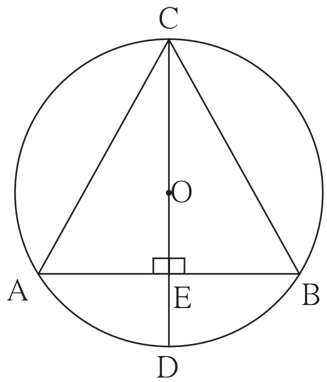


Fig. 6.21

ICT Tools or Links

Draw different circles with Geogebra software. Verify and experience the properties of chords. Draw circumcircle and incircle of different triangles. Using 'Move Option' experience how the incentre and circumcentre changes if the size of a triangle is changed.

❑❑❑

7 Co-ordinate Geometry

Lets study

- Axis, Origin, Quadrant
- Co-ordinates of a point in a plane.
- To plot a point.
- Line parallel to X-axis.
- Line parallel to Y-axis.
- Equation of a line.

Chintu and his friends were playing cricket on the ground in front of a big building, when a visitor arrived.

Visitor : Hey Chintu, Dattabhau lives here, doesnt he ?

Chintu : Yes, on the second floor. See that window ? Thats his flat.

Visitor : But there are five windows on the second floor. It could be any of them !

Chintu : His window is the third one from the left, on the second floor..

Chintu's description of the location of Dattabhau's flat is in fact, based on the most basic concept in Co-ordinate Geometry.

It did not suffice to give only the floor number to locate the house. Its serial number from the left or from the right also needed to be given. That is two numbers had to be given in a specific sequence. Two **ordinal numbers** namely, **second** from the ground and **third** from the left had to be used.

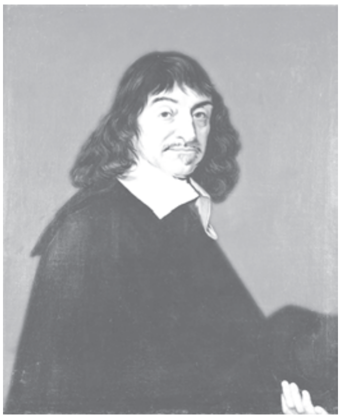
Let' Learn

Axes, origin, quadrants

We could give the location of Dattabhau's house using two ordinal numbers. Similarly, the location of a point can be fully described using its distances from two mutually perpendicular lines.

To locate a point in a plane, a horizontal number line is drawn in the plane. This number line is called the X-axis.

Rene Descartes (1596-1650)



Rene Descartes, a French mathematician of the 17th Century, proposed the co-ordinate system to describe the position of a point in a plane accurately. It is called the Cartesian co-ordinate system. Obviously the word Cartesian is derived from his name. He brought about a revolution in the field of mathematics by establishing the relationship between Algebra and Geometry.

The Cartesian co-ordinate system is the foundation of Analytical Geometry. La Geometric was Descartes' first book on mathematics. In it, he used algebra for the study of geometry and proposed that a point in a plane can be represented by an ordered pair of real numbers. This ordered pair is the 'Cartesian Co-ordinates' of a point.

Co-ordinate geometry has used in a variety of fields such as Physics, Engineering, Nautical Science, Siesmology and Art. It plays an important role in the development of technology in Geogebra. We see the inter-relationship between Algebra and Geometry quite clearly in the software Geogebra;the very name being a combination of the words 'Geometry' and 'Algebra'.

Another number line intersecting the X-axis at point marked O and perpendicular to the X-axis, is the Y-axis. Generally, the number O is represented by the same point on both the number lines. This point is called the origin and is shown by the letter O.

On the X-axis, positive numbers are shown on the right of O and negative numbers on the left.

On the Y-axis, positive numbers are shown above O and negative numbers below it.

The X and Y axes divide the plane into four parts, each of which is called a **Quadrant.** As shown in the figure, the quadrants are numbered in the anti-clocksise direction.

The points on the axes are not included in the quadrants.

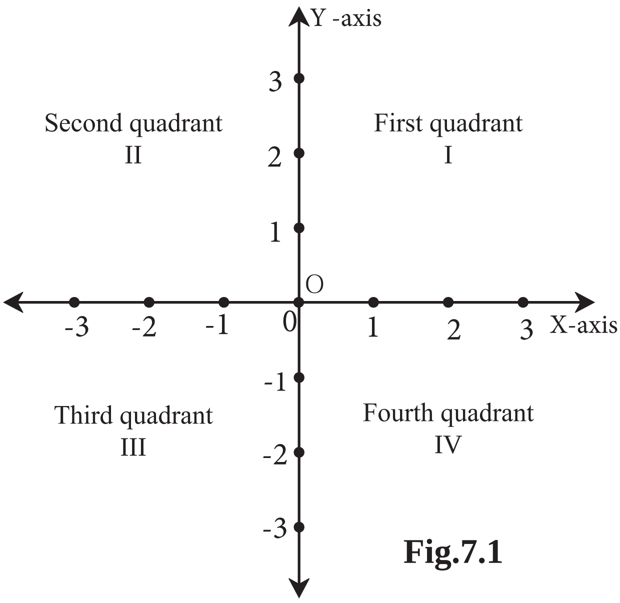


Fig.7.1

The Co-ordinates of a point in a plane

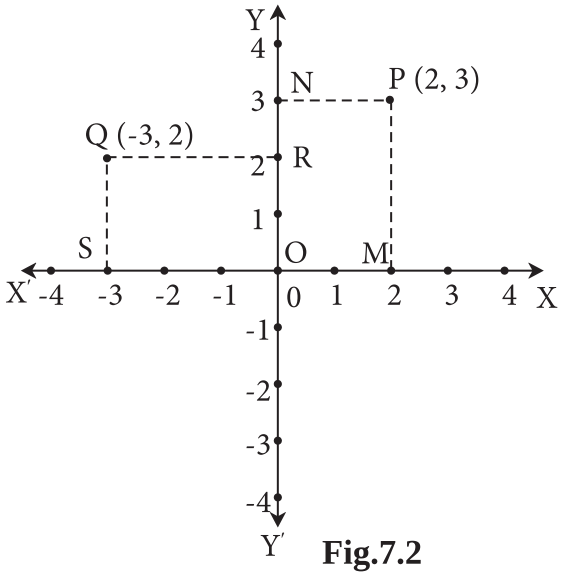


Fig.7.2

The point P is shown in the plane determined by the X-axis and the Y-axis. Its position can be determined by its distance from the two axes. To find these distances, we draw seg PM ⊥ X-axis and seg PN ⊥ Y-axis.

Co-ordinate of point M on X-axis is 2 and co-ordinate of point N on Y-axis is 3.

Therefore *x* co-ordinate of point P is 2 and *y* co-ordinate of point P is 3..

The convention for describing the position of a point is to mention *x* co-ordinate first. According to this convention the order of co-ordinates of point P is decided as 2, 3. The position of the point P in brief, is described by the pair (2, 3)

The order of the numbers in the pair (2, 3) is important. Such a pair of numbers is called an ordered pair.

To describe the position of point Q, we draw seg QS ⊥ X-axis and seg QR ⊥ Y-axis. The co-ordinate of point Q on the X-axis is –3 and the co-ordinate on the Y-axis is 2. Hence the co-ordinates of point Q are (–3, 2).

Ex. Write the co-ordinates of points E, F, G, T in the figure alongside.

Solution :

- The co-ordinates of point E are (2, 1)
- The co-ordinates of point F are (–3, 3)
- The co-ordinates of point G are (–4, –2).
- The co-ordinates of point T are (3, –1)

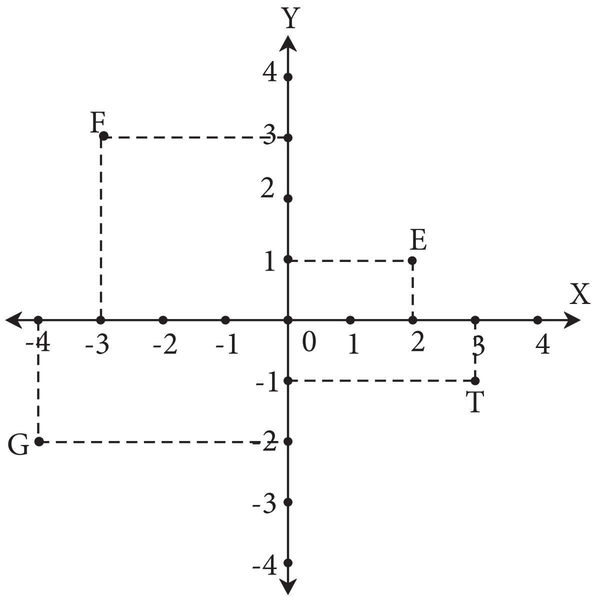


Fig.7.3

Let's learn.

Co-ordinates of points on the axes

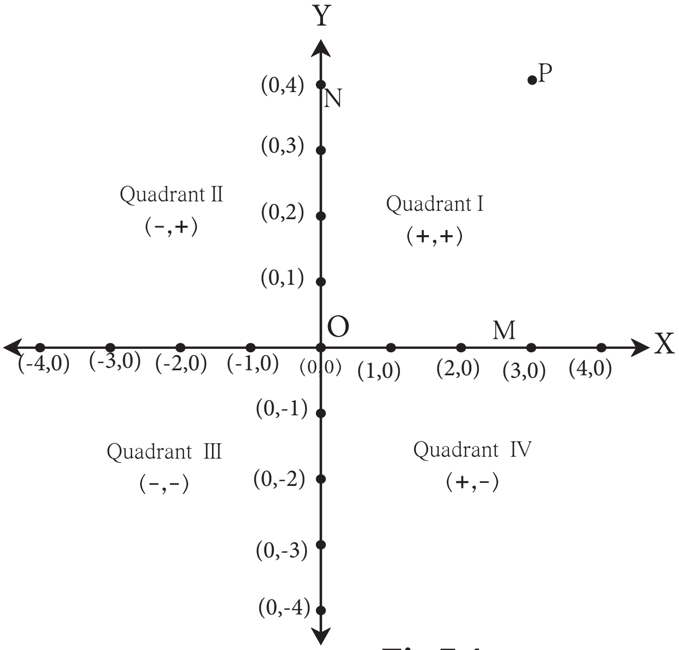


Fig.7.4

The *x* co-ordinate of point M is its distance from the Y-axis. The distance of point M from the X-axis is zero. Hence, the *y* co-ordinate of M is 0.

Thus, the co-ordinates of point M on the X-axis are (3,0).

The *y* co-ordinate of point N on the Y-axis is 4 units from the X-axis because N is at a distance of 4. Its *x* co-ordinate is 0 because its distance from the Y-axis is zero.

Hence, the co-ordinates of point N on the Y-axis are (0, 4).

Now the origin 'O' is on X-axis as well as on Y-axis. Hence, its distance from X-axis and Y-axis is zero. Therefore, the co-ordinates of O are (0, 0).

One and only one pair of co-ordinates (ordered pair) is associated with every point in a plane.

Let's Remember

- The *y* co-ordinate of every point on the X-axis is zero.
- The *x* co-ordinate of every point on the Y-axis is zero.
- The coordinates of the origin are (0, 0).

Ex. In which quadrant or on which axis are the points given below ?
A(5,7), B(–6,4), C(4,–7), D(–8,–9), P(–3,0), Q(0,8)

Solution : The *x* co-ordinate of A (5, 7) is positive and its *y* co-ordinate is positive..
∴ point A is in the first quadrant.
The *x* co-ordinate of B (–6, 4) is negative and *y* co-ordinate is positive.
∴ point B is in the second quadrant.
The *x* co-ordinate of C (4, –7) is positive and *y* co-ordinate is negative.
∴ point C is in the fourth quadrant.
The *x* co-ordinate of D (–8, –7) is negative and *y* co-ordinate is negative.
∴ point D is in the third quadrant.

The *y* co-ordinate of P(-3,0) is zero ∴ point P is on the X-axis.

The *x* co-ordinate of Q (0,8) is zero ∴ point Q is on the Y-axis.

Activity As shown in fig. 7.5, ask girls to sit in lines so as to form the X-axis and Y-axis.

- Ask some boys to sit at the positions marked by the coloured dots in the four quadrants.
- Now, call the students turn by turn using the initial letter of each student's name. As his or her initial is called, the student stands and gives his or her own co-ordinates. For example Rajendra (2, 2) and Kirti (-1, 0)
- Even as they have fun during this field activity, the students will learn how to state the position of a point in a plane.

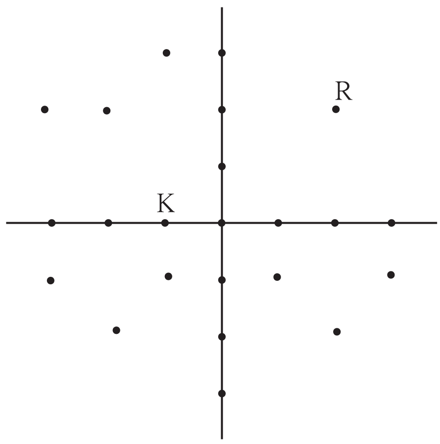


Fig. 7.5

Let's learn.

To plot the points of given co-ordinates

Suppose we have to plot the points P (4,3) and Q (-2,2)

Steps for plotting the points

(i) Draw X-axis and Y-axis on the plane. Show the origin.

(ii) To find the point P (4,3), draw a line parallel to the Y-axis through the point on X axis which represents the number 4.

Through the point on Y-axis which represents the number 3 draw a line parallel to the X-axis .

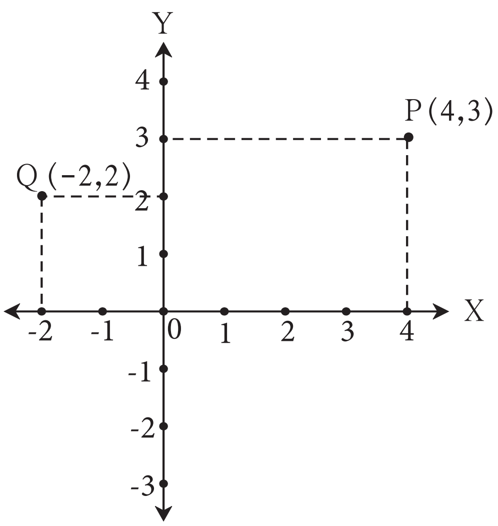


Fig. 7.6

(iii) The point of intersection of these two lines parallel to the Y and X-axis respectively, is the point P (4,3). In which quadrant does this point lie ?

(iv) In the same way, plot the point Q (–2, 2) . Is this point in the second quadrant ? Using the same method, plot the points R(–3, –4), S(3, –1)

Ex. In which quadrants or on which axis are the points given below ?

(i) (5, 3) (ii) (–2, 4) (iii) (2, –5) (iv) (0, 4)

(v) (–3, 0) (vi) (–2, 2.5) (vii) (5, 3.5) (viii) (–3.5, 1.5)

(ix) (0, –4) (x) (2, –4)

Solution :

	co-ordinates	Quadrant / axis
(i)	(5,3)	Quadrant I
(ii)	(–2,4)	Quadrant II
(iii)	(2,–5)	Quadrant IV
(iv)	(0,4)	Y–axis
(v)	(–3,0)	X–axis

	co-ordinates	Quadrant / axis
(vi)	(–2, –2.5)	Quadrant III
(vii)	(5,3.5)	Quadrant I
(viii)	(–3.5,1.5)	Quadrant II
(ix)	(0, –4)	Y–axis
(x)	(2,–4)	Quadrant IV

Practice set 7.1

1. State in which quadrant or on which axis do the following points lie.

- A(–3, 2), • B(–5, –2), • K(3.5, 1.5), • D(2, 10),
- E(37, 35), • F(15, –18), • G(3, –7), • H(0, –5),
- M(12, 0), • N(0, 9), • P(0, 2.5), • Q(–7, –3)

2. In which quadrant are the following points ?

(i) whose both co-ordinates are positive.

(ii) whose both co-ordinates are negative.

(iii) whose x co-ordinate is positive, and the y co-ordinate is negative.

(iv) whose x co-ordinate is negative and y co-ordinate is positive.

3. Draw the co-ordinate system on a plane and plot the following points.

L(–2, 4), M(5, 6), N(–3, –4), P(2, –3), Q(6, –5), S(7, 0), T(0, –5)

Let's learn.

Lines parallel to the X-axis

- On a graph paper, plot the following points
  A (5, 4), B (2, 4), C (–2, 4), D (–4, 4), E (0, 4), F (3, 4)
- Observe the co-ordinates of the given points.
- Did you notice that the y co-ordinates of all the points are equal ?
- All the points are collinear.
- To which axis is this line parallel ?
- The y co-ordinate of every point on the line DA is 4. It is constant. Therefore the line DA is described by the equation $y = 4$. If the y co-ordinate of any point is 4, will be on the line DA.

  The equation of the line parallel to the X axis at a distance of 4 units from the X-axis is $y = 4$.

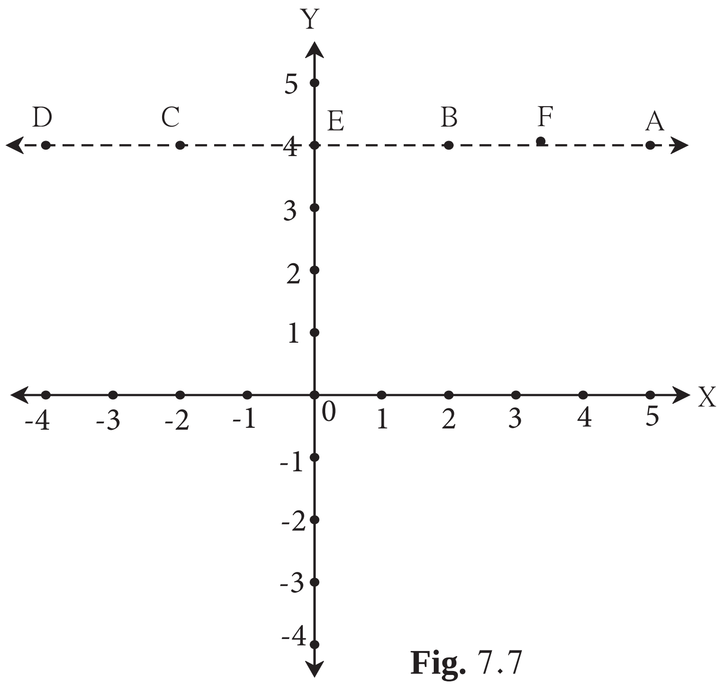


Fig. 7.7

Let's discuss.

- Can we draw a line parallel to the X-axis at a distance of 6 units from it and below the X-axis ?
- Will all of the points (–3, –6), (10, –6), ($\frac{1}{2}$, –6) be on that line ?
- What would be the equation of this line ?

Remember this !

If b > 0, and we draw the line y = b through the point (0, b), it will be above the X-axis and parallel, to it. If b < 0, then the line $y = b$ will be below the X-axis and parallel to it.

The equation of a line parallel to the X-axis is in the form $y = b$.

Let's learn.

Lines parallel to the Y-axis

- On a graph paper, plot the following points
  P(–4, 3), Q(–4, 0), R(–4, 1), S(–4, –2), T(–4, 2), U(–4, –3)

- Observe the co–ordinates of the points.
- Did you notice that the x co–ordinate of all the points are the same ?
- Are all the points collinear ?
- To which axis is this line parallel ?
- The x co–ordinate of every point on the line PS is -4. It is constant. Therefore, the line PS can be described by the equation $x = -4$. Every point whose x co–ordinate is -4 lies on the line PS. The equation of the line parallel to the Y–axis at a distance of 4 units and to the left of Y–axis is $y = -4$.

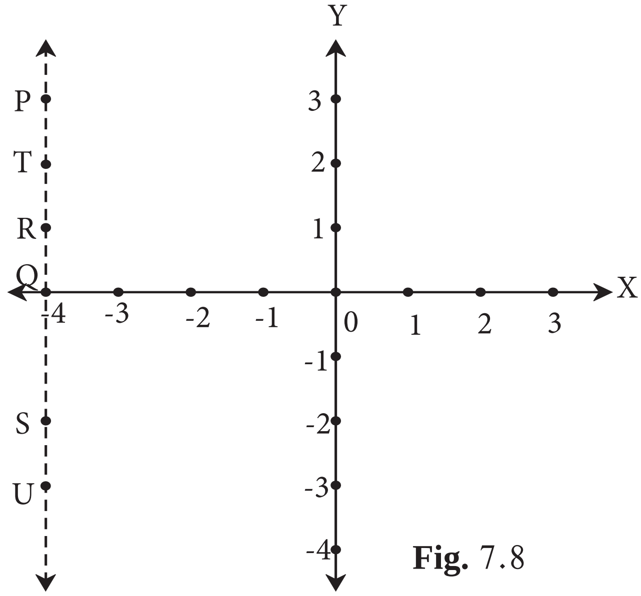


Fig. 7.8

Let's discuss.

- Can we draw a line parallel to the Y-axis at a distance of 2 units from it and to its right ?
- Will all of the points (2,10), (2,8), $(2, -\frac{1}{2})$ be on that line ?
- What would be the equation of this line ?

Remember this !

If we draw the line $x = a$ parallel to the Y-axis passing through the point $(a, 0)$ and if $a > 0$ then the line will be to the right of the Y-axis. If $a < 0$, then the line will be to the left of the Y-axis.

The equation of a line parallel to the Y-axis is in the form $x = a$.

Remember this !

(1) The y co-ordinate of every point on the X-axis is zero. Conversely, every point whose y co-ordinate is zero is on the X-axis. Therefore, the equation of the X axis is $y = 0$.

(2) The x co-ordinate of every point on the Y-axis is zero. Conversely, every point whose x co-ordinate is zero is on the Y-axis. Therefore, the equation of the Y-axis is $x = 0$.

Let's learn.

Graph of a linear equations

Ex. Draw the graphs of the equations $x = 2$ and $y = -3$.

Solution : (i) On a graph paper draw the X-axis and the Y-axis.

(ii) Since it is given that $x = 2$, draw a line on the right of the Y-axis at a distance of 2 units from it and parallel to it.

(iii) Since it is given that $y = -3$, draw a line below the X-axis at a distance of 3 units from it and parallel to it.

(iv) These lines, parallel to the two axes, are the graphs of the given equations.

(v) Write the co-ordinates of the point P, the point of intersection of these two lines.

(vi) Verify that the co-ordinates of the point P are $(2, -3)$

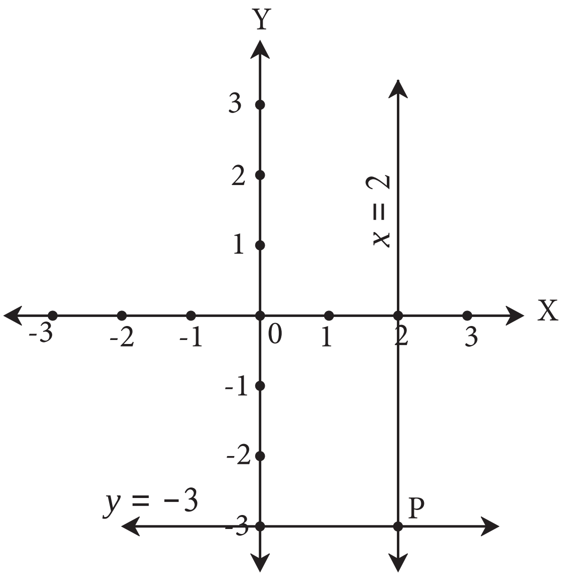


Fig. 7.9

The graph of a linear equation in the general form.

Activity : On a graph paper, plot the points (0,1) (1,3) (2,5). Are they collinear ? If so, draw the line that passes through them.

- Through which quadrants does this line pass ?
- Write the co-ordinates of the point at which it intersects the Y-axis.
- Show any point in the third quadrant which lies on this line. Write the co-ordinates of the point.

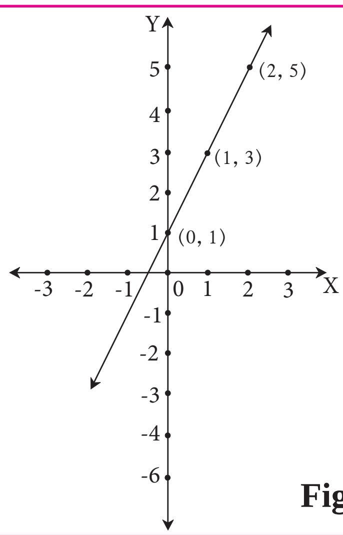


Fig. 7.10

Ex. $2x - y + 1 = 0$ is a linear equation in two variables in general form. Let us draw the graph of this equation.

Solution : $2x - y + 1 = 0$ means $y = 2x + 1$

Let us assume some values of x and find the corresponding values of y.

For example, If $x = 0$, then substituting this value of x in the equation we get $y = 1$.

Similarly, let us find the values of y when 0, 1, 2, $\frac{1}{2}$, -2 are some values of x and write these values in the table below in the form of ordered pairs.

x	0	1	2	$\frac{1}{2}$	-2
y	1	3	5	2	-3
(x, y)	(0,1)	(1,3)	(2,5)	$(\frac{1}{2}, 2)$	$(-2, -3)$

Now, let us plot these points. Let us verify that these points are collinear. Let us draw that line. The line is the graph of the equaiton $2x - y + 1 = 0$.

ICT Tools or Links

Use the Software Geogebra to draw the X and Y-axis. Plot several points. Find and study the co-ordinates of the points in 'Algebraic view'. Read the equations of lines that are parallel to the axes. Use the 'move' option to vary the positions of the lines. What are the equations of the X-axis and the Y-axis ?

Practice set 7.2

1. On a graph paper plot the points A (3,0), B(3,3), C(0,3). Join A, B and B, C. What is the figure formed ?

2. Write the equation of the line parallel to the Y-axis at a distance of 7 units from it to its left.

3. Write the equation of the line parallel to the X-axis at a distance of 5 units from it and below the X-axis.

4. The point Q(-3, -2) lies on a line parallel to the Y-axis. Write the equation of the line and draw its graph.

5. X-axis and line $x = -4$ are parallel lines. What is the distance between them?

6. Which of the equations given below have graphs parallel to the X-axis, and which ones have graphs parallel to the Y-axis ?

(i) $x = 3$ (ii) $y - 2 = 0$ (iii) $x + 6 = 0$ (iv) $y = -5$

7. On a graph paper, plot the points A(2, 3), B(6, –1) and C(0, 5). If those points are collinear then draw the line which includes them. Write the co-ordinates of the points at which the line intersects the X-axis and the Y-axis.

8. Draw the graphs of the following equations on the same system of co-ordinates. Write the co-ordinates of their points of intersection.

$x + 4 = 0$, $y - 1 = 0$, $2x + 3 = 0$, $3y - 15 = 0$

9. Draw the graphs of the equations given below

(i) $x + y = 2$ (ii) $3x - y = 0$ (iii) $2x + y = 1$

Problem set 7

1. Choose the correct alternative answer for the following quesitons.

(i) What is the form of co-ordinates of a point on the X-axis ?

(A) (b, b) (B) $(0, b)$ (C) $(a, 0)$ (D) (a, a)

(ii) Any point on the line $y = x$ is of the form .....

(A) (a, a) (B) $(0, a)$ (C) $(a, 0)$ (D) $(a, -a)$

(iii) What is the equation of the X-axis ?

(A) $x = 0$ (B) $y = 0$ (C) $x + y = 0$ (D) $x = y$

(iv) In which quadrant does the point (–4, –3) lie ?

(A) First (B) Second (C) Third (D) Fourth

(v) What is the nature of the line which includes the points (–5,5), (6,5), (–3,5), (0,5) ?

(A) Passes through the origin,, (B) Parallel to Y–axis.

(C) Parallel to X–axis (D) None of these

(vi) Which of the points P (–1,1), Q (3,–4), R(1,–1), S (–2,–3), T (–4,4) lie in the fourth quadrant ?

(A) P and T (B) Q and R (C) only S (D) P and R

2. Some points are shown in the figure 7.11

With the help of it answer the following questions :

(i) Write the co-ordinates of the points Q and R.

(ii) Write the co-ordinates of the points T and M.

(iii) Which point lies in the third quadrant ?

(iv) Which are the points whose *x* and *y* co-ordinates are equal ?

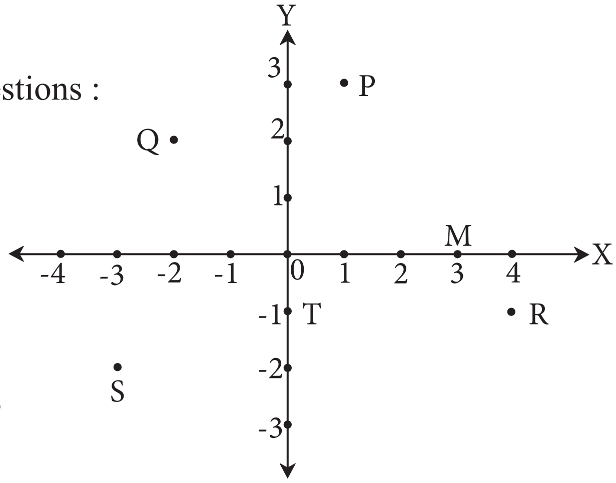


Fig.7.11

3. Without plotting the points on a graph, state in which quadrant or on which axis do the following point lie.

(i) (5, –3) (ii) (–7, –12) (iii) (–23, 4)

(iv) (–9, 5) (v) (0, –3) (vi) (–6, 0)

4. Plot the following points on the one and the same co-ordinate system.

A(1, 3), B(–3, –1), C(1, –4),
D(–2, 3), E(0, –8), F(1, 0)

5. In the graph alongside, line LM is parallel to the Y-axis. (Fig. 7.12)

(i) What is the distance of line LM from the Y-axis ?

(ii) Write the co-ordinates of the points P, Q and R.

(iii) What is the difference between the *x* co-ordinates of the points L and M?

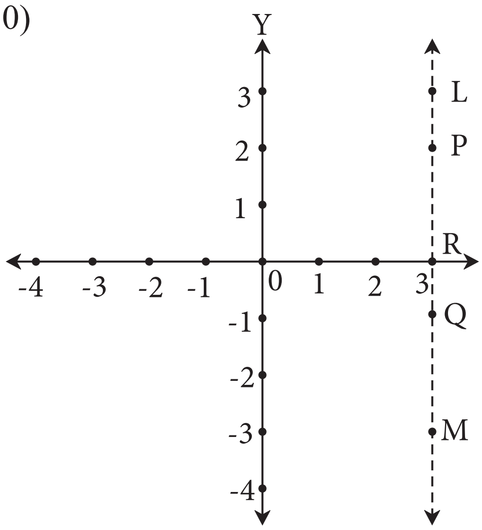


Fig.7.12

6. How many lines are there which are parallel to X-axis and having a distance 5 units?

7*. If '*a*' is a real number, what is the distance between the Y-axis and the line $x = a$?

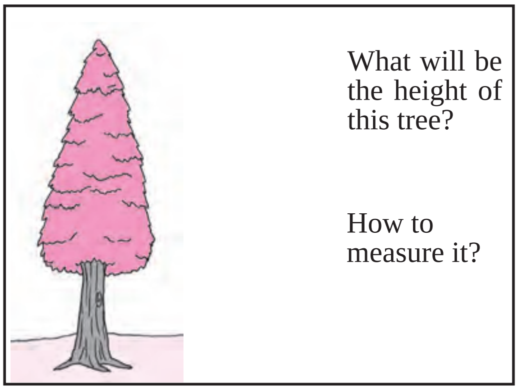
8 Trigonometry

24XROH

Let's study.

- **Introduction of Trigonometry**
- **Trigonometic Ratios**
- **Relations among Trigonometric Ratios**
- **Trigonometric Ratios of Particular Angles**

Introduction to Trigonometry



We can measure distances by using a rope or by walking on ground, but how to measure the distance between a ship and a light house? How to measure the height of a tall tree?

Observe the above pictures. Questions in the pictures are related to mathematics. Trigonometry, a branch of mathematics, is useful to find answers to such questions. Trigonometry is used in different branches of Engineering, Astronomy, Navigation etc.

The word Trigonometry is derived from three Greek words 'Tri' means three, 'gona' means sides and 'metron' means measurements.

Let's recall.

We have studied triangle. The subject trigonometry starts with right angled triangle, theorem of Pythagoras and similar triangles, so we will recall these topics.

- In Δ ABC, $\angle$B is a right angle and side AC opposite to $\angle$B, is hypotenuse. Side opposite to $\angle$A is BC and side opposite to $\angle$C is AB.
  Using Pythagoras' theorem, we can write the following statement for this triangle.
  $(AB)^2 + (BC)^2 = (AC)^2$

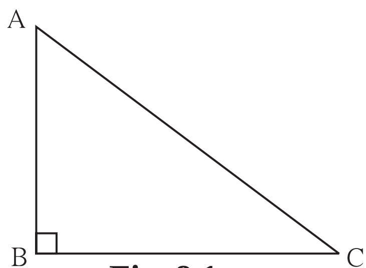


Fig. 8.1

- If $\Delta ABC \sim \Delta PQR$ then their corresponding sides are in the same proportions.

  So $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

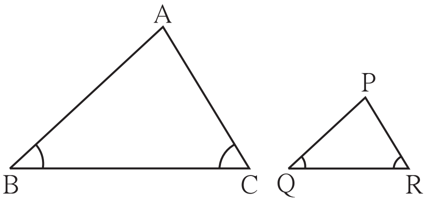


Fig. 8.2

Let us see how to find the height of a tall tree using properties of similar triangles.

Activity : This experiment can be conducted on a clear sunny day.

Look at the figure given alongside.

Height of the tree is QR, height of the stick is BC.

Thrust a stick in the ground as shown in the figure. Measure its height and length of its shadow. Also measure the length of the shadow of the tree. Rays of sunlight are parallel. So Δ PQR and Δ ABC are equiangular, means similar triangles. Sides of similar triangles are proportional.

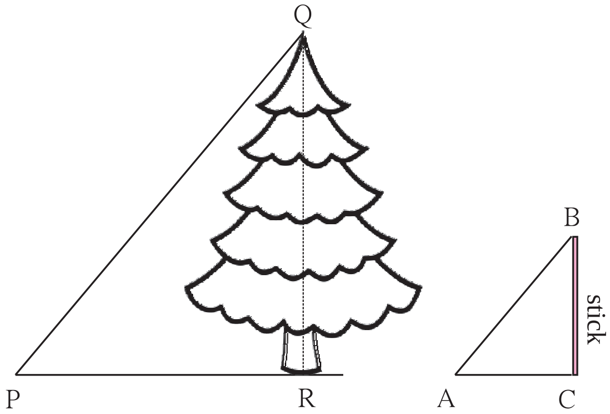


Fig.8.3

So we get $\frac{QR}{PR} = \frac{BC}{AC}$.

Therefore, we get an equation,

height of the tree = QR = $\frac{BC}{AC} \times PR$

We know the values of PR, BC and AC. Substituting these values in this equation, we get length of QR, means height of the tree.

Use your brain power !

It is convenient to do this experiment between 11:30 am and 1:30 pm instead of doing it in the morning at 8'O clock. Can you tell why ?

Activity : You can conduct this activity and find the height of a tall tree in your surrounding. If there is no tree in the premises then find the height of a pole.

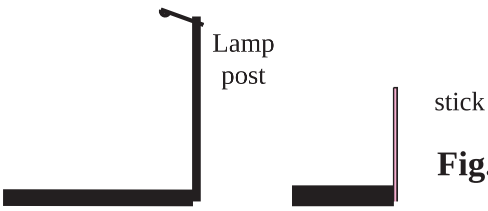


Fig. 8.4

Let's learn.

Terms related to right angled triangle

In right angled Δ ABC, $\angle$B = 90°, $\angle$ A and $\angle$ C are acute angles.

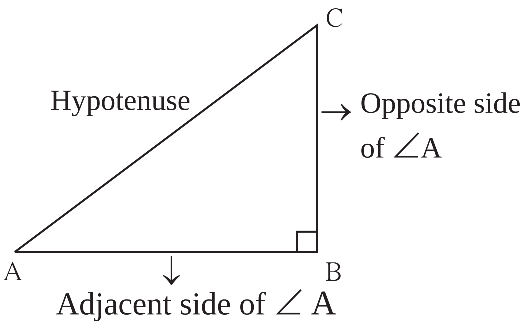


Fig. 8.5

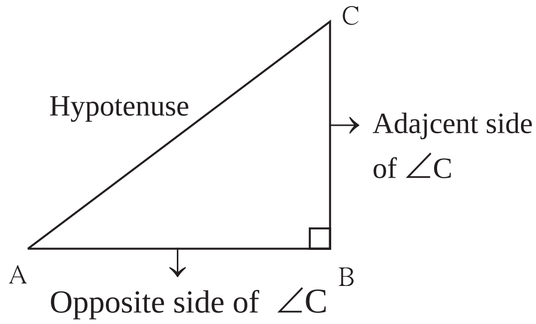


Fig. 8.6

Ex. In the figure 8.7, Δ PQR is a right angled triangle. Write-

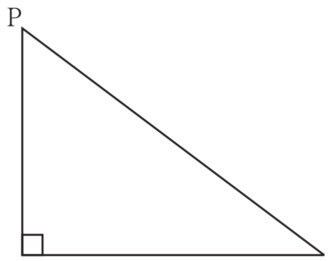


Fig. 8.7

side opposite to $\angle$ P =.....

side opposite to $\angle$ R =.....

side adjacent to $\angle$ P = ...

side adjacent to $\angle$ R = ...

Trigonometic ratios

In the adjacent Fig.8.8 some right angled triangles are shown. $\angle$B is their common angle. So all right angled triangles are similar.

Δ PQB ~ Δ ACB

$$\therefore \frac{PB}{AB} = \frac{PQ}{AC} = \frac{BQ}{BC}$$

$$\therefore \frac{PQ}{AC} = \frac{PB}{AB} \quad \therefore \frac{PQ}{PB} = \frac{AC}{AB} \quad \text{... alternando}$$

$$\frac{QB}{BC} = \frac{PB}{AB} \quad \therefore \frac{QB}{PB} = \frac{BC}{AB} \quad \text{... alternando}$$

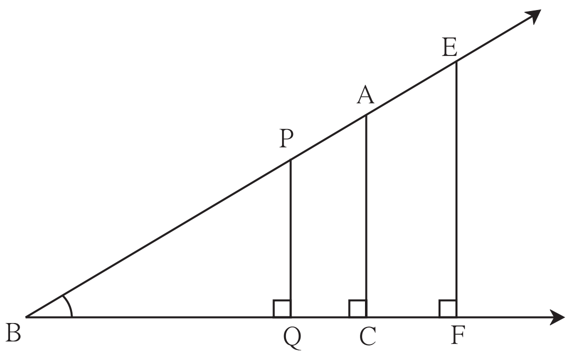


Fig. 8.8

The figures of triangles in 8.9 and.8.10 are of the triangles separated from the figure 8.8

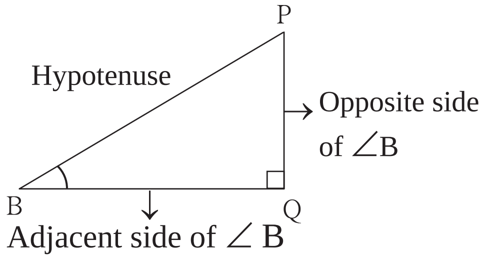


Fig.8.9

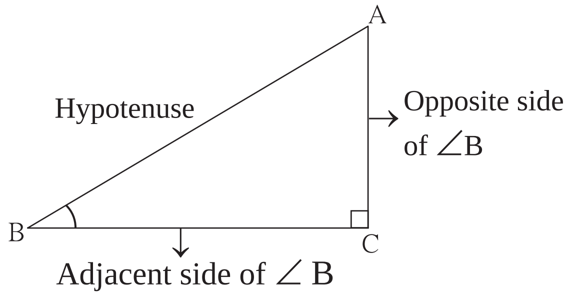


Fig.8.10

(i) In Δ PQB,

$$\frac{PQ}{PB} = \frac{\text{Opposite side of } \angle B}{\text{Hypotenuse}}$$

In Δ ACB,

$$\frac{AC}{AB} = \frac{\text{Opposite side of } \angle B}{\text{Hypotenuse}}$$

The ratios $\frac{PQ}{PB}$ and $\frac{AC}{AB}$ are equal.

$$\therefore \frac{PQ}{PB} = \frac{AC}{AB} = \frac{\text{Opposite side of } \angle B}{\text{Hypotenuse}}$$

This ratio is called the 'sine' ratio of $\angle$ B, and is written in brief as **'sin B'**.

(ii) In Δ PQB and Δ ACB,

$$\frac{BQ}{PB} = \frac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}} \quad \text{and} \quad \frac{BC}{AB} = \frac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}}$$

$$\therefore \frac{BQ}{PB} = \frac{BC}{AB} = \frac{\text{Adjacent side of } \angle B}{\text{Hypotenuse}}$$

This ratio is called as the 'cosine' ratio of $\angle$ B , and written in brief as **'cos B'**

(iii) $\frac{PQ}{BQ} = \frac{AC}{BC} = \frac{\text{Opposite side of } \angle B}{\text{Adjacent side of } \angle B}$

This ratio is called as the tangent ratio of $\angle$ B, and written in brief as **tan B.**

Ex. :

Fig.8.11

Sometimes we write measures of acute angles of a right angled triangle by using Greek letters θ (Theta), α (Alpha), β (Beta) etc.

In the adjacent figure of Δ ABC, measure of acute angle C is denoted by the letter θ. So we can write the ratios sin C, cos C, tan C as sin θ, cos θ, tan θ respectively.

$\sin C = \sin\theta = \frac{AB}{AC}$, $\cos C = \cos\theta = \frac{BC}{AC}$, $\tan C = \tan\theta = \frac{AB}{BC}$

Remember this !

- sin ratio = $\frac{\text{opposite side}}{\text{hypotenuse}}$
- cos ratio = $\frac{\text{adjacent side}}{\text{hypotenuse}}$
- tan ratio = $\frac{\text{opposite side}}{\text{adjcent side}}$
- $\sin\theta = \frac{\text{opposite side of } \angle\theta}{\text{hypotenuse}}$
- $\cos\theta = \frac{\text{adjacent side of } \angle\theta}{\text{hypotenuse}}$
- $\tan\theta = \frac{\text{opposite side of } \angle\theta}{\text{opposite side of } \angle\theta}$

Practice set 8.1

1.

Fig. 8.12

In the Fig.8.12, $\angle R$ is the right angle of Δ PQR. Write the following ratios.
(i) sin P (ii) cos Q (iii) tan P (iv) tan Q

2.

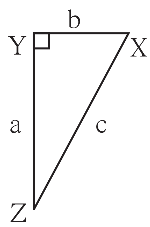


Fig. 8.13

In the right angled Δ XYZ, $\angle XYZ = 90^\circ$ and *a*,*b*,*c* are the lengths of the sides as shown in the figure. Write the following ratios,
(i) sin X (ii) tan Z (iii) cos X (iv) tan X.

3.

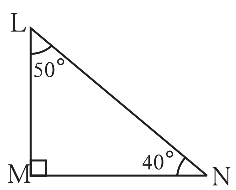


Fig. 8.14

In right angled Δ LMN, $\angle LMN = 90^\circ$
$\angle L = 50^\circ$ and $\angle N = 40^\circ$,
write the following ratios.
(i) sin 50° (ii) cos 50°
(iii) tan 40° (iv) cos 40°

4.

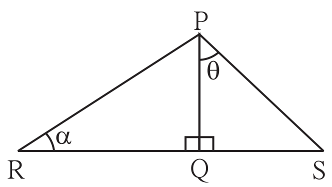


Fig. 8.15

In the figure 8.15,.$\angle PQR = 90^\circ$,
$\angle PQS = 90^\circ$, $\angle PRQ = \alpha$ and $\angle QPS = \theta$
Write the following trigonometric ratios.
(i) $\sin\alpha$, $\cos\alpha$, $\tan\alpha$
(ii) $\sin\theta$, $\cos\theta$, $\tan\theta$

Let's learn.

Relation among trigonometric ratios

In the Fig.8.16

Δ PMN is a right angled triangle.

$\angle$ M = 90°, $\angle$P and $\angle$ N are complimentary angles.

$\therefore$ If $\angle$ N = θ then $\angle$ P = 90 – θ

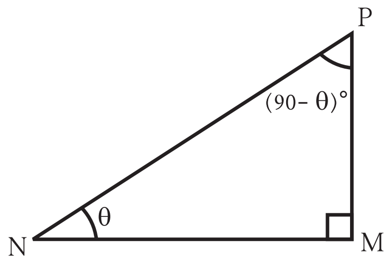


Fig.8.16

$\sin\theta = \dfrac{PM}{PN}$ ....(1)

$\cos\theta = \dfrac{NM}{PN}$ ....(2)

$\tan\theta = \dfrac{PM}{NM}$ ....(3)

$\sin(90-\theta) = \dfrac{NM}{PN}$ ....(4)

$\cos(90-\theta) = \dfrac{PM}{PN}$ ....(5)

$\tan(90-\theta) = \dfrac{NM}{PM}$ ....(6)

$\therefore$ $\sin\theta = \cos(90-\theta)$ .... from (1) and (5)

$\cos\theta = \sin(90-\theta)$ .... from (2) and (4)

Also note that $\tan\theta \times \tan(90-\theta) = \dfrac{PM}{NM} \times \dfrac{NM}{PM}$ .... from (3) and (6)

$\therefore$ $\tan\theta \times \tan(90-\theta) = 1$

Similarly, $\dfrac{\sin\theta}{\cos\theta} = \dfrac{\frac{PM}{PN}}{\frac{NM}{PN}} = \dfrac{PM}{PN} \times \dfrac{PN}{NM} = \dfrac{PM}{NM} = \tan\theta$

Remember this !

$\cos(90-\theta) = \sin\theta$, $\sin(90-\theta) = \cos\theta$

$\dfrac{\sin\theta}{\cos\theta} = \tan\theta$, $\tan\theta \times \tan(90-\theta) = 1$

*** For more information**

$$\frac{1}{\sin\theta} = \text{cosec}\,\theta, \quad \frac{1}{\cos\theta} = \sec\theta, \quad \frac{1}{\tan\theta} = \cot\theta$$

It means cosec θ, sec θ and cot θ are inverse ratios of sin θ, cos θ and tan θ respectively.

- $\sec\theta = \text{cosec}\,(90-\theta)$
- $\text{cosec}\,\theta = \sec(90-\theta)$
- $\tan\theta = \cot(90-\theta)$
- $\cot\theta = \tan(90-\theta)$

Let's recall.

Theorem of 30°- 60°-90° triangle :

We know that if the measures of angles of a triangle are 30°,60°, 90° then side opposite to 30° angle is half of the hypotenuse and side opposite to 60° angle is $\frac{\sqrt{3}}{2}$ of hypotenuse.

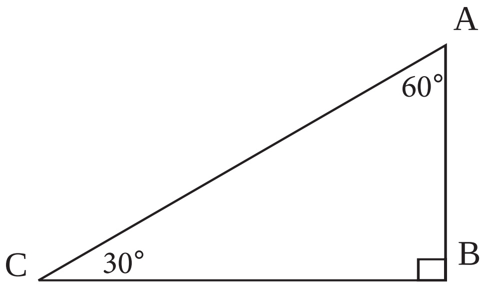


Fig. 8.17

In the Fig. 8.17, Δ ABC is a right angled triangle. ∠C = 30°, ∠A = 60°, ∠B = 90° .

$$\therefore\ AB = \frac{1}{2}AC \text{ and } BC = \frac{\sqrt{3}}{2}AC$$

Let's learn.

Trignometric ratios of 30° and 60° angles

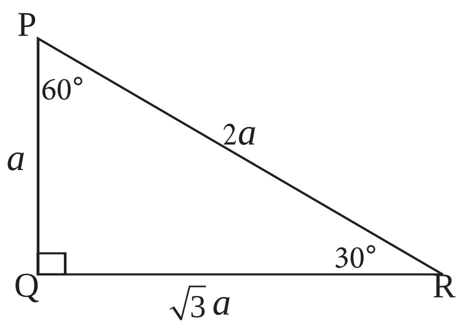


Fig. 8.18

In right angled Δ PQR if ∠R = 30°, ∠P = 60°, ∠Q = 90° and PQ = a

then $PQ = \frac{1}{2}PR$

$a = \frac{1}{2}PR$

$\therefore\quad PR = 2a$

$QR = \frac{\sqrt{3}}{2}PR$

$QR = \frac{\sqrt{3}}{2} \times 2a$

$QR = \sqrt{3}\,a$

∴ If PQ = a, then PR = $2a$ and QR = $\sqrt{3}\,a$

(I) Trigonometric ratios of the 30° angle

$$\sin 30^\circ = \frac{PQ}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{QR}{PR} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{PQ}{QR} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

(II) Trigonometric ratios of 60° angle

$$\sin 60^\circ = \frac{QR}{PR} = \frac{\sqrt{3}\,a}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{PQ}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{QR}{PQ} = \frac{\sqrt{3}\,a}{a} = \sqrt{3}$$

In right angled Δ PQR, $\angle Q = 90^\circ$. Therefore $\angle P$ and $\angle R$ are complimentary angles of each other. Verify the relation between sine and cosine ratios of complimentary angles here also.

$\sin \theta = \cos (90 - \theta)$
$\sin 30^\circ = \cos (90^\circ - 30^\circ) = \cos 60^\circ$
$\sin 30^\circ = \cos 60^\circ$

$\cos \theta = \sin (90 - \theta)$
$\cos 30^\circ = \sin (90^\circ - 30^\circ) = \sin 60^\circ$
$\cos 30^\circ = \sin 60^\circ$

Remember this !

$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{1}{\sqrt{3}}$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \sqrt{3}$

(III) Trigonometric ratios of the 45° angle

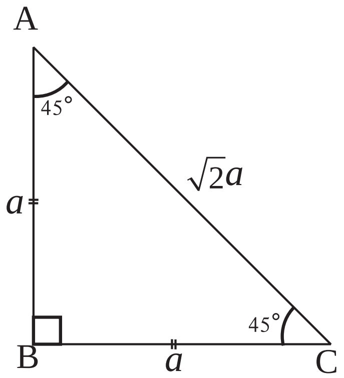


Fig.8.19

In right angled Δ ABC, $\angle B = 90^\circ$, $\angle A = 45^\circ$, $\angle C = 45^\circ$ $\therefore$ This is an isosceles triangle.
Suppose AB = a then BC = a.
Using Pythagoras' theorem,
let us find the length of AC.

$$AC^2 = AB^2 + BC^2$$
$$= a^2 + a^2$$
$$AC^2 = 2a^2$$
$$\therefore \ AC = \sqrt{2}\,a$$

In the Fig. 8.19, $\angle C = 45°$

$$\sin 45° = \frac{AB}{AC} = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}}$$

$$\cos 45° = \frac{BC}{AC} = \frac{a}{\sqrt{2}\,a} = \frac{1}{\sqrt{2}}$$

$$\tan 45° = \frac{AB}{BC} = \frac{a}{a} = 1$$

Remember this !

$\sin 45° = \frac{1}{\sqrt{2}}$, $\cos 45° = \frac{1}{\sqrt{2}}$, $\tan 45° = 1$

(IV) Trigonometric ratios of the angle 0° and 90°

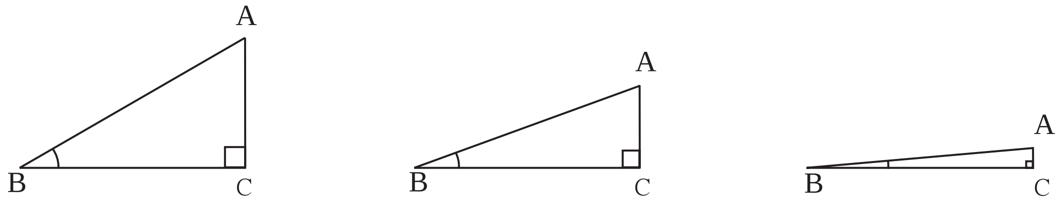


Fig.8.20

In the right angled Δ ACB, $\angle C = 90°$ and $\angle B = 30°$. We know that $\sin 30° = \frac{AC}{AB}$. Keeping the length of side AB constant, if the measure of $\angle B$ goes on decreasing the length of AC, which is opposite to $\angle B$ also goes on decreasing. So as the measure of $\angle B$ decreases, then value of $\sin \theta$ also decreases.

$\therefore$ when measure of $\angle B$ becomes 0°, then length of AC becomes 0.

$\therefore \sin 0° = \frac{AC}{AB} = \frac{0}{AB} = 0 \quad \therefore \sin 0° = 0$

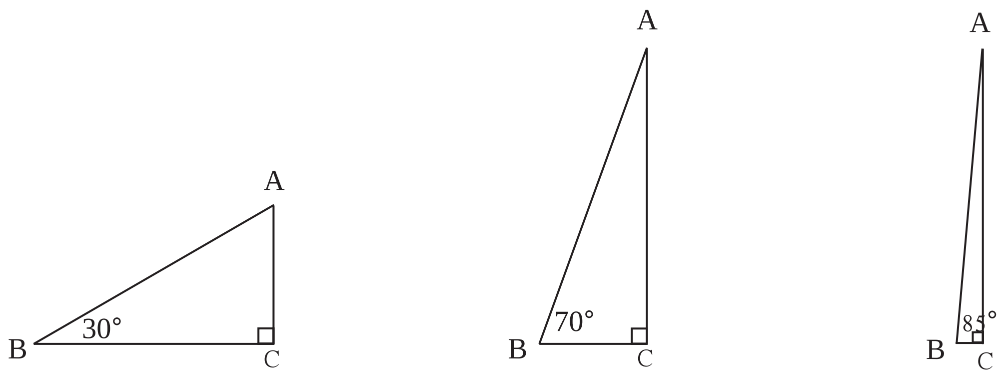


Fig.8.21

Now look at the Fig. 8.21. In this right angled triangle, as the measure of $\angle$B increases the length of AC also increases. When measure of $\angle$B becomes 90°, the length of AC become equal to AB

$\therefore \sin 90^\circ = \frac{AC}{AB}$ $\qquad\qquad$ $\therefore \sin 90^\circ = 1$

We know the relations between trigonometric ratios of complimentary angles.

$$\sin \theta = \cos (90 - \theta) \quad \text{and} \quad \cos \theta = \sin (90 - \theta)$$

$$\therefore \cos 0^\circ = \sin (90 - 0)^\circ = \sin 90^\circ = 1$$

$$\text{and } \cos 90^\circ = \sin (90 - 90)^\circ = \sin 0^\circ = 0$$

Remember this !

$\sin 0^\circ = 0,$ $\qquad$ $\sin 90^\circ = 1,$ $\qquad$ $\cos 0^\circ = 1,$ $\qquad$ $\cos 90^\circ = 0$

We know that,

$\tan \theta = \frac{\sin \theta}{\cos \theta}$ $\qquad\qquad$ $\therefore \tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = 0$

But $\tan 90^\circ = \frac{\sin 90^\circ}{\cos 90^\circ} = \frac{1}{0}$

But we can not do the division of 1 by 0. Note that θ is an acute angle. As it increases and reaches the value of 90°, tan θ also increases indefinitely. Hence we can not find the definite value of tan 90.

Remember this !

Trigonometric ratios of particular ratios.

Ratios \ Measures of angles	0°	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Undefined

Solved Examples :

Ex. (1) Find the value of 2tan 45° + cos 30° – sin 60°

Solution : 2tan 45° + cos 30° – sin 60°

$= 2 \times 1 + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$

$= 2 + 0$

$= 2$

Ex. (2) Find the value of $\frac{\cos 56^\circ}{\sin 34^\circ}$

Solution : $56^\circ + 34^\circ = 90^\circ$ means 56 and 34 are the measures of complimentary angles.

$\sin \theta = \cos (90- \theta)$

$\therefore$ $\sin 34^\circ = \cos (90- 34)^\circ = \cos 56^\circ$

$\therefore$ $\frac{\cos 56^\circ}{\sin 34^\circ} = \frac{\cos 56^\circ}{\cos 56^\circ} = 1$

Ex. 3 In right angled Δ ACB, If $\angle C = 90^\circ$, AC = 3, BC = 4.

Find the ratios sin A, sin B, cos A, tan B

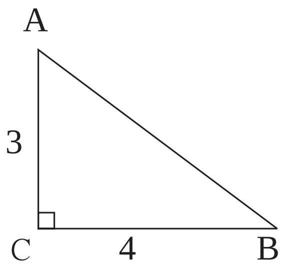


Fig. 8.22

Solution : In right angled Δ ACB, using Pythagoras' theorem,

$AB^2 = AC^2 + BC^2$

$= 3^2 + 4^2 = 5^2$

$\therefore$ AB = 5

$\sin A = \frac{BC}{AB} = \frac{4}{5}$ $\qquad$ $\cos A = \frac{AC}{AB} = \frac{3}{5}$

and $\sin B = \frac{AC}{AB} = \frac{3}{5}$ $\qquad$ $\tan B = \frac{AC}{BC} = \frac{3}{4}$

Ex. 4 In right angled triangle Δ PQR, $\angle Q = 90^\circ$, $\angle R = \theta$ and if $\sin \theta = \frac{5}{13}$ then find $\cos \theta$ and $\tan \theta$.

Solution :

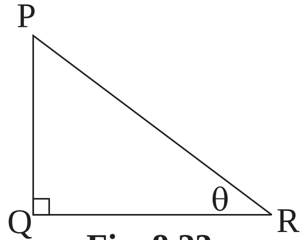


Fig. 8.23

In right angled Δ PQR, $\angle R = \theta$

$\sin \theta = \frac{5}{13}$

$\therefore$ $\frac{PQ}{PR} = \frac{5}{13}$

$\therefore$ Let PQ = $5k$ and PR = $13k$

Let us find QR by using Pythagoras' theorem,

$$PQ^2 + QR^2 = PR^2$$
$$(5k)^2 + QR^2 = (13k)^2$$
$$25k^2 + QR^2 = 169\,k^2$$
$$QR^2 = 169\,k^2 - 25k^2$$
$$QR^2 = 144\,k^2$$
$$QR = 12k$$

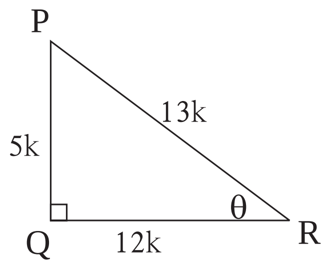


Fig. 8.24

Now, in right angled Δ PQR, PQ = $5k$, PR = $13k$ and QR = $12k$

$$\therefore \cos\theta = \frac{QR}{PR} = \frac{12k}{13k} = \frac{12}{13}, \ \tan\theta = \frac{PQ}{QR} = \frac{5k}{12k} = \frac{5}{12}$$

Use your brain power!

(1) While solving above example, why the lengths of PQ and PR are taken $5k$ and $13k$?

(2) Can we take the lengths of PQ and PR as 5 and 13 ? If so then what changes are needed in the writing of the solution.

Important Equation in Trigonometry

Δ PQR is a right angled triangle.

$\angle PQR = 90^\circ$, $\angle R = \theta$

$$\sin\theta = \frac{PQ}{PR} \text{ ....(I)}$$

and $$\cos\theta = \frac{QR}{PR} \text{ ....(II)}$$

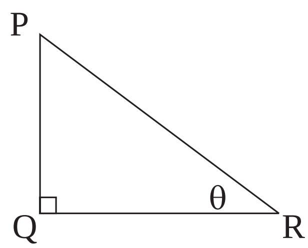


Fig.8.25

Using Pythagoras' theorem,

$PQ^2 + QR^2 = PR^2$

$$\therefore \frac{PQ^2}{PR^2} + \frac{QR^2}{PR^2} = \frac{PR^2}{PR^2}$$ .... dividing each term by PR^2

$$\therefore \left(\frac{PQ}{PR}\right)^2 + \left(\frac{QR}{PR}\right)^2 = 1$$

$\therefore$ $(\sin\theta)^2 + (\cos\theta)^2 = 1$ ..from (I) & (II)

Remember this !

'Square of' $\sin\theta$ means $(\sin\theta)^2$. It is written as $\sin^2\theta$.

We have proved the equation $\sin^2\theta + \cos^2\theta = 1$ using Pythagoras' theorem, where θ is an acute angle of a right angled triangle.

Verify that the equation is true even when $\theta = 0°$ or $\theta = 90°$

Since the equation $\sin^2\theta + \cos^2\theta = 1$ is true for any value of θ. So it is a basic trigonometrical identity.

(i) $0 \le \sin\theta \le 1, \quad 0 \le \sin^2\theta \le 1$ (ii) $0 \le \cos\theta \le 1, \quad 0 \le \cos^2\theta \le 1$

Practice set 8.2

1. In the following table, a ratio is given in each column. Find the remaining two ratios in the column and complete the table.

$\sin\theta$		$\frac{11}{61}$		$\frac{1}{2}$				$\frac{3}{5}$	
$\cos\theta$	$\frac{35}{37}$				$\frac{1}{\sqrt{3}}$				
$\tan\theta$			1			$\frac{21}{20}$	$\frac{8}{15}$		$\frac{1}{2\sqrt{2}}$

2. Find the values of -

(i) $5\sin 30° + 3\tan 45°$

(ii) $\frac{4}{5}\tan^2 60° + 3\sin^2 60°$

(iii) $2\sin 30° + \cos 0° + 3\sin 90°$

(iv) $\frac{\tan 60}{\sin 60 + \cos 60}$

(v) $\cos^2 45° + \sin^2 30°$

(vi) $\cos 60° \times \cos 30° + \sin 60° \times \sin 30°$

3. If $\sin\theta = \frac{4}{5}$ then find $\cos\theta$

4. If $\cos\theta = \frac{15}{17}$ then find $\sin\theta$

Problem set 8

1. Choose the correct alternative answer for following multiple choice questions.

(i) Which of the following statements is true ?

(A) $\sin\theta = \cos(90-\theta)$ (B) $\cos\theta = \tan(90-\theta)$

(C) $\sin\theta = \tan(90-\theta)$ (D) $\tan\theta = \tan(90-\theta)$

(ii) Which of the following is the value of $\sin 90^\circ$?

(A) $\frac{\sqrt{3}}{2}$ (B) 0 (C) $\frac{1}{2}$ (D) 1

(iii) $2\tan 45^\circ + \cos 45^\circ - \sin 45^\circ =$?

(A) 0 (B) 1 (C) 2 (D) 3

(iv) $\frac{\cos 28^\circ}{\sin 62^\circ} =$?

(A) 2 (B) -1 (C) 0 (D) 1

2. In right angled Δ TSU, TS = 5, $\angle S = 90^\circ$, SU = 12 then find sin T, cos T, tan T. Similarly find sin U, cos U, tan U.

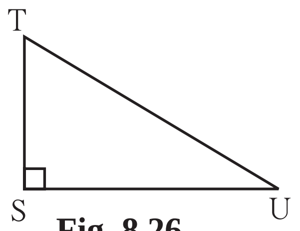


Fig. 8.26

3. In right angled Δ YXZ, $\angle X = 90^\circ$, XZ = 8 cm, YZ = 17 cm, find sin Y, cos Y, tan Y, sin Z, cos Z, tan Z.

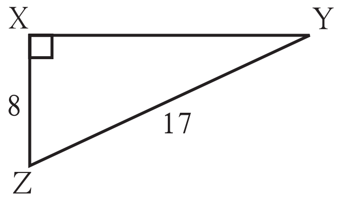


Fig. 8.27

4. In right angled Δ LMN, if $\angle N = \theta$, $\angle M = 90^\circ$, $\cos\theta = \frac{24}{25}$, find $\sin\theta$ and $\tan\theta$

Similarly, find $(\sin^2\theta)$ and $(\cos^2\theta)$.

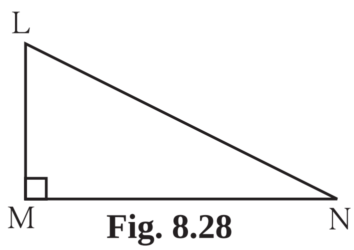


Fig. 8.28

5. Fill in the blanks.

(i) $\sin 20^\circ = \cos \square^\circ$

(ii) $\tan 30^\circ \times \tan \square^\circ = 1$

(iii) $\cos 40^\circ = \sin \square^\circ$

9 Surface Area and Volume

Let's study.

- Surface area of a cone
- Volume of a cone
- Surface area of a sphere
- Volume of a sphere

Let's recall.

We have learnt how to find the surface area and volume of a cuboid, a cube and a cylinder, in earlier standard.

Cuboid

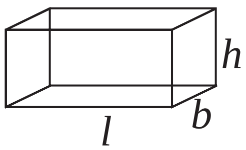


Fig.9.1

- Length, breadth and height of a cuboid are l , b, h respectively.

  (i) Area of vertical surfaces of a cuboid = $2(l + b) \times h$

  Here we have considered only 4 surfaces into consideration.

  (ii) Total surface area of a cuboid = $2(lb + bh + lh)$

  Here we have taken all 6 surfaces into consideration.

  (iii) Volume of a cuboid = $l \times b \times h$

Cube

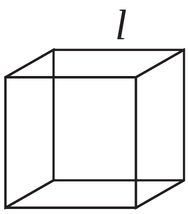


Fig.9.2

- If l is the edge of a cube,

  (i) Total surface area of a cube = $6l^2$

  (ii) Area of vertical surfaces of a cube = $4l^2$

  (iii) Volume of a cube = l^3

Cylinder

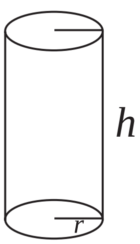


Fig.9.3

- Radius of cylinder is r and height is h.

  (i) Curved surface area of a cylinder = $2\pi rh$

  (ii) Total surface area of a cylinder = $2\pi r(r + h)$

  (iii) Volume of a cylinder = πr^2h

Practice set 9.1

1. Length, breadth and height of a cuboid shape box of medicine is 20cm, 12 cm and 10 cm respectively. Find the surface area of vertical faces and total surface area of this box.
2. Total surface area of a box of cuboid shape is 500 sq. unit. Its breadth and height is 6 unit and 5 unit respectively. What is the length of that box ?
3. Side of a cube is 4.5 cm. Find the surface area of all vertical faces and total surface area of the cube.
4. Total surface area of a cube is 5400 sq. cm. Find the surface area of all vertical faces of the cube.
5. Volume of a cuboid is 34.50 cubic metre. Breadth and height of the cuboid is 1.5m and 1.15m respectively. Find its length.
6. What will be the volume of a cube having length of edge 7.5 cm ?
7. Radius of base of a cylinder is 20cm and its height is 13cm, find its curved surface area and total surface area. ($\pi = 3.14$)
8. Curved surface area of a cylinder is 1980 cm^2 and radius of its base is 15cm. Find the height of the cylinder. ($\pi = \frac{22}{7}$).

Let's learn.

Terms related to a cone and their relation

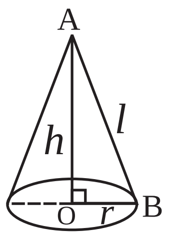


Fig.9.4

A cone is shown in the adjacent Fig.9.4. Centre of the circle, which is the base of the cone, is O and A is the vertex (apex) of the cone. Seg OB is a a radius and seg OA is perpendicular to the radius at O, means AO is perpendicular height of the cone. Slant height of the cone is the length of AB, which is shown by (l).

Δ AOB is a right angled triangle.

$\therefore$ by the Pythagoras' theorem

$$AB^2 = AO^2 + OB^2$$

$$\therefore l^2 = h^2 + r^2$$

That is, (slant height)^2 = (Perpendicular height)^2 + (Base radius)^2

Surface area of a cone

A cone has two surfaces : (i) circular base and (ii) curved surface.

Out of these two we can find the area of base of a cone because we know the formula for the area of a circle.

How to find the curved surface area of a cone ? How to derive a formula for it ?

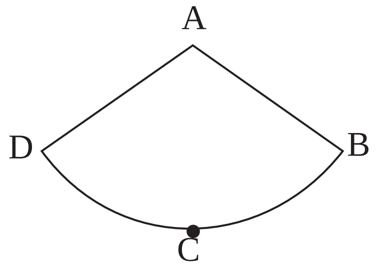


Fig.9.5

To find a formula for the curved surface area of a cone, let us see the net of the curved surface, which is a sector of a circle.

If a cone is cut along edge AB,we get its net as shown in fig.9.5.

Compare the figures 9.4 and 9.5

Have you noticed the following things ?

(i) Radius AB of the sector is the same as the slant height of the cones.

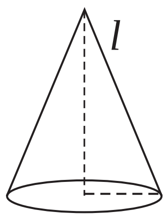
(ii) Arc BCD of the sector is the same as circumference of the base of the cone.

(iii) Curved surface area of cone = Area of sector A-BCD.

It means to find the curved surface area of a cone we have to find the area of its net that is the area of the sector.

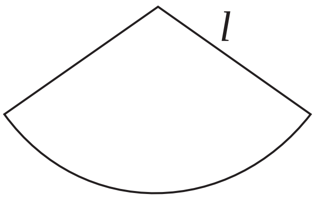
Try to understand, how it is done from the following activity.

Activity : Look at the following figures.



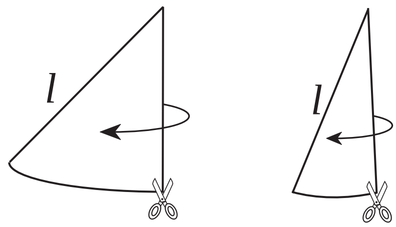
Cone

Fig. 9.6



Net of curved surface

Fig. 9.7



Pieces of the net

Fig. 9.8

Circumference of base of the circle $= 2\pi r$

As shown in the Fig.9.8, make pieces of the net as small as possible. Join them as shown in the Fig.9.9.

By Joining the small pieces of net of the cone, we get a rectangle ABCD approximately.

Total length of AB and CD is $2\pi r$.

$\therefore$ length of side AB of rectangle ABCD is πr and length of side CD is also πr.

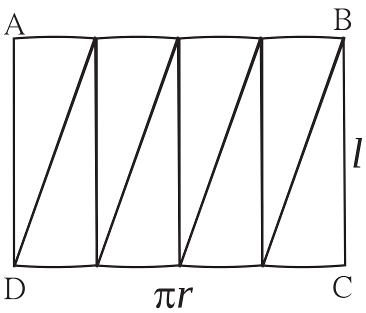


Fig. 9.9

Length of side BC of rectangle = slant height of cone $= l$.

Curved surface area of cone is equal to the area of the rectangle.

$\therefore$ curved surface area of cone = Area of rectangle $= AB \times BC = \pi r \times l = \pi rl$

Now, we can derive the formula for total surface area of a cone.

Total surface area of cone = Curved surface area + Area of base

$$= \pi rl + \pi r^2$$

$$= \pi r(l + r)$$

Did you notice a thing ? If a cone is not closed (Just like a cap of jocker or a cap in a birthday party) it will have only one surface, which is the curved surface. Then we get the surface area of the cone by the formula πrl.

Activity : Prepare a cylinder of a card sheet, keeping one of its faces open. Prepare an open cone of card sheet which will have the same base-radius and the same height as that of the cylinder.

Pour fine sand in the cone till it just fills up the cone. Empty the cone in the cylinder. Repeat the procedure till the cylinder is just filled up with sand. Note how many coneful of sand is required to fill up the cylinder.

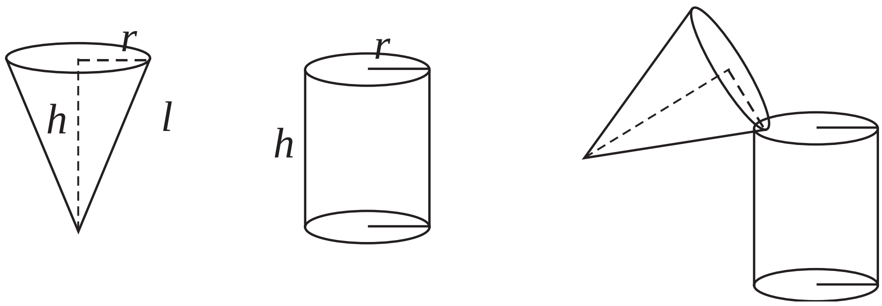


Fig. 9.10

To fill up the cylinder, three coneful of sand is required.

Let's learn.

Volume of a cone

If the base-radii and heights of a cone and a cylinder are equal then

$$3 \times \text{volume of cone} = \text{volume of cylinder}$$

$$\therefore\ 3 \times \text{volume of cone} = \pi r^2 h$$

$$\therefore\ \text{volume of cone} = \frac{1}{3} \times \pi r^2 h$$

Remember this !

(i) Area of base of a cone = πr^2

(ii) Curved surface area of a cone = πrl

(iii) Total surface area of a cone = $\pi r\,(l + r)$

(iv) Volume of a cone = $\frac{1}{3} \times \pi r^2 h$

Solved Examples :

Ex. (1) Radius of base (r) and perpendicular height (h) of cone is given. Find its slant height (l)

(i) r = 6 cm, h = 8 cm, (ii) r = 9 cm, h = 12 cm

Solution :

(i) $r = 6$ cm, $h = 8$ cm

$l^2 = r^2 + h^2$

$\therefore l^2 = (6)^2 + (8)^2$

$\therefore l^2 = 36 + 64$

$\therefore l^2 = 100$

$\therefore l = 10$ cm

(ii) $r = 9$ cm, $h = 12$ cm

$l^2 = r^2 + h^2$

$\therefore l^2 = (9)^2 + (12)^2$

$\therefore l^2 = 81 + 144$

$\therefore l^2 = 225$

$\therefore l = 15$ cm

Ex. (2) Find (i) the slant height, (ii) the curved surface area and (iii) total surface area of a cone, if its base radius is 12 cm and height is 16 cm. (π = 3.14)

Solution :

(i) $r = 12$ cm, $h = 16$ cm

$l^2 = r^2 + h^2$

$\therefore l^2 = (12)^2 + (16)^2$

$\therefore l^2 = 144 + 256$

$\therefore l^2 = 400$

$\therefore l = 20$ cm

(ii) Curved surface area $= \pi r l$

$= 3.14 \times 12 \times 20$

$= 753.6 \text{ cm}^2$

(iii) Total surface area of cone

$= \pi r (l + r)$

$= 3.14 \times 12(20+12)$

$= 3.14 \times 12 \times 32$

$= 1205.76 \text{ cm}^2$

Ex. (3) The total surface area of a cone is 704 sq.cm and radius of its base is 7 cm, find the slant height of the cone. ($\pi = \frac{22}{7}$)

Solution : Total surface area of cone $= \pi r (l + r)$

$\therefore \quad 704 = \frac{22}{7} \times 7 (l + 7)$

$\therefore \quad \frac{704}{22} = l + 7$

$\therefore \quad 32 = l + 7$

$\therefore \quad 32 - 7 = l$

$\therefore \quad l = 25$ cm

Ex. (4) Area of the base of a cone is 1386 sq.cm and its height is 28 cm.

Find its surface area. ($\pi = \frac{22}{7}$)

Solution :

Area of base of cone $= \pi r^2$

$\therefore \quad 1386 = \frac{22}{7} \times r^2$

$\therefore \quad \frac{1386 \times 7}{22} = r^2$

$\therefore \quad 63 \times 7 = r^2$

$\therefore \quad 441 = r^2$

$\therefore \quad r = 21$ cm

$\therefore l^2 = (21)^2 + (28)^2$

$\therefore l^2 = 441 + 784$

$\therefore l^2 = 1225$

$\therefore l = 35$ cm

Surface area of cone $= \pi rl$

$= \frac{22}{7} \times 21 \times 35$

$= 22 \times 21 \times 5$

$= 2310$ sq. cm.

Practice set 9.2

1. Perpendicular height of a cone is 12 cm and its slant height is 13 cm. Find the radius of the base of the cone.
2. Find the volume of a cone, if its total surface area is 7128 sq.cm and radius of base is 28 cm. ($\pi = \frac{22}{7}$)
3. Curved surface area of a cone is 251.2 cm^2 and radius of its base is 8cm. Find its slant height and perpendicular height. ($\pi = 3.14$)
4. What will be the cost of making a closed cone of tin sheet having radius of base 6 m and slant height 8 m if the rate of making is Rs.10 per sq.m ?
5. Volume of a cone is 6280 cubic cm and base radius of the cone is 30 cm. Find its perpendicular height. ($\pi = 3.14$)
6. Surface area of a cone is 188.4 sq.cm and its slant height is 10cm. Find its perpendicular height ($\pi = 3.14$)
7. Volume of a cone is 1212 cm^3 and its height is 24cm. Find the surface area of the cone. ($\pi = \frac{22}{7}$)
8. The curved surface area of a cone is 2200 sq.cm and its slant height is 50 cm. Find the total surface area of cone. ($\pi = \frac{22}{7}$)
9. There are 25 persons in a tent which is conical in shape. Every person needs an area of 4 sq.m. of the ground inside the tent. If height of the tent is 18m, find the volume of the tent.

10. In a field, dry fodder for the cattle is heaped in a conical shape. The height of the cone is 2.1m. and diameter of base is 7.2 m. Find the volume of the fodder. if it is to be covered by polythin in rainy season then how much minimum polythin sheet is needed ?

$(\pi = \frac{22}{7}$ and $\sqrt{17.37} = 4.17.)$

Let's learn.

Surface area of a sphere

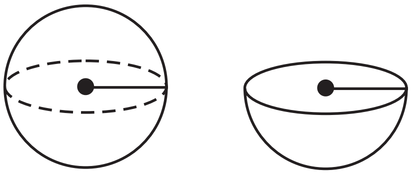


Fig. 9.11

Surface area of a sphere = $4\pi r^2$

$\therefore$ Surface area of a hollow hemisphere = $2\pi r^2$

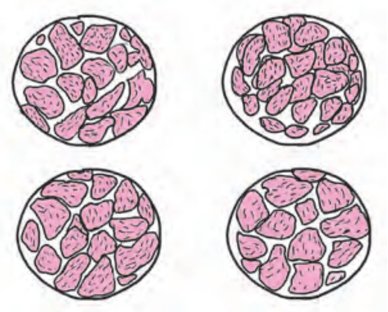
Total surface area of a solid hemisphere

= Surface area of hemisphere + Area of circle

= $2\pi r^2 + \pi r^2 = 3\pi r^2$

Take a sweet lime (Mosambe), Cut it into two equal parts.

Take one of the parts. Place its circular face on a paper. Draw its circular border. Copy three more such circles. Again, cut each half of the sweet lime into two equal parts.



Now you get 4 quarters of sweet lime. Separate the peel of a quarter part. Cut it into pieces as small as possible. Try to cover one of the circles drawn, by the small pieces.

Observe that the circle gets nearly covered.

The activity suggests that,

curved surface area of a sphere = $4\pi r^2$.

Solved Examples :

(1) Find the surface area of a sphere having radius 7 cm. ($\pi = \frac{22}{7}$)

Solution : Surface Area of sphere = $4\pi r^2$

$$= 4 \times \frac{22}{7} \times (7)^2$$

$$= 4 \times \frac{22}{7} \times 7 \times 7$$

$$= 88 \times 7$$

$$= 616$$

Surface Area of sphere = 616 sq.cm.

(2) Find the radius of a sphere having surface area 1256sq.cm.(π = 3.14)

Solution : Surface Area of Sphere = $4\pi r^2$

$\therefore\ 1256 = 4 \times 3.14 \times r^2$

$\therefore\ r^2 = \frac{1256}{4 \times 3.14}$

$= \frac{31400}{314}$

$\therefore\ 100 = r^2$

$\therefore\ 10 = r$

$\therefore$ radius of the sphere is 10 cm.

Activity : Make a cone and a hemisphere of cardsheet such that radii of cone and hemisphere are equal and height of cone is equal to radius of the hemisphere. Fill the cone with fine sand. Pour the sand in the hemisphere. How many cones are required to fill the hemisphere completely ?

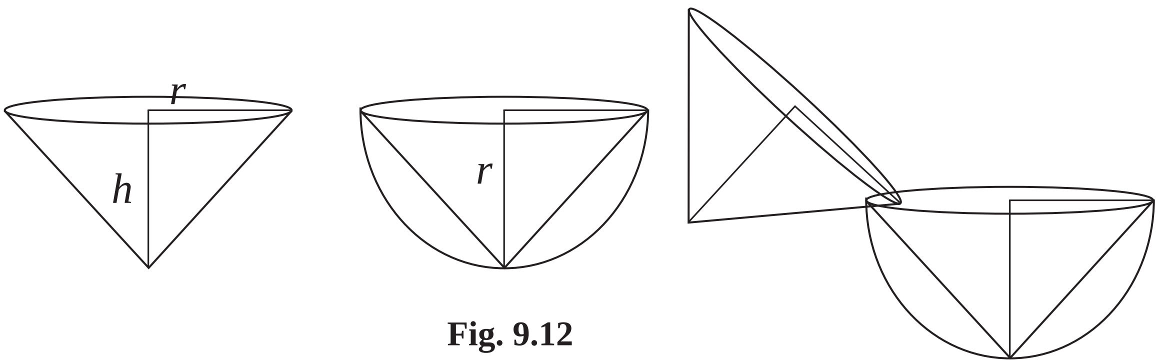


Fig. 9.12

Two conefull of sand is required to fill the hemisphere.

$\therefore\ 2 \times$ volume of cone = volume of hemisphere.

$\therefore$ volume of hemisphere = $2 \times$ volume of cone

$$= 2 \times \frac{1}{3} \times \pi r^2 h$$

$$= 2 \times \frac{1}{3} \times \pi r^2 \times r$$

$$= \frac{2}{3}\pi r^3$$

$\therefore$ volume of sphere

$= 2 \times$ volume of hemisphere.

$= \frac{4}{3}\pi r^3$

$\therefore$ volume of sphere = $\frac{4}{3}\pi r^3$

Remember this !

- Volume of hemisphere = $\frac{2}{3}\pi r^3$
- Total surface area of hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

Solved Examples :

Ex. (1) Find the volume of a sphere having radius 21 cm. ($\pi = \frac{22}{7}$)

Solution : Volume of sphere = $\frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times \frac{22}{7} \times (21)^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$$

$$= 88 \times 441$$

$\therefore$ volume of sphere = 38808 cubic cm.

Ex. (2) Find the radius of a sphere whose volume is 113040 cubic cm. ($\pi = 3.14$)

Solution : Volume of sphere = $\frac{4}{3}\pi r^3$

$$113040 = \frac{4}{3} \times 3.14 \times r^3$$

$$\frac{113040 \times 3}{4 \times 3.14} = r^3$$

$$\frac{28260 \times 3}{3.14} = r^3$$

$$\therefore 9000 \times 3 = r^3$$

$$\therefore r^3 = 27000$$

$$\therefore r = 30 \text{ cm}$$

$\therefore$ radius of sphere is 30 cm.

Ex. (3) Find the volume of a sphere whose surface area is 314.sq.cm. (Take $\pi = 3.14$)

Solution : Surface area of sphere = $4\pi r^2$

$$314 = 4 \times 3.14 \times r^2$$

$$\frac{314}{4 \times 3.14} = r^2$$

$$\frac{31400}{4 \times 314} = r^2$$

$$\therefore \frac{100}{4} = r^2$$

$$\therefore 25 = r^2$$

$$\therefore r = 5 \text{ cm}$$

Volume of sphere $= \frac{4}{3}\pi r^3$

$$= \frac{4}{3} \times 3.14 \times 5^3$$

$$= \frac{4}{3} \times 3.14 \times 125$$

$$= 523.33 \text{ cubic cm.}$$

Practice set 9.3

1. Find the surface areas and volumes of spheres of the following radii.
   (i) 4 cm (ii) 9 cm (iii) 3.5 cm. ($\pi = 3.14$)
2. If the radius of a solid hemisphere is 5cm, then find its curved surface area and total surface area. ($\pi = 3.14$)
3. If the surface area of a sphere is 2826 cm^2 then find its volume. ($\pi = 3.14$)
4. Find the surface area of a sphere, if its volume is 38808 cubic cm. ($\pi = \frac{22}{7}$)
5. Volume of a hemisphere is 18000 π cubic cm. Find its diameter.

Problem set 9

1. If diameter of a road roller is 0.9 m and its length is 1.4 m, how much area of a field will be pressed in its 500 rotations ?
2. To make an open fish tank, a glass sheet of 2 mm gauge is used. The outer length, breadth and height of the tank are 60.4 cm, 40.4 cm and 40.2 cm respectively. How much maximum volume of water will be contained in it ?
3. If the ratio of radius of base and height of a cone is 5:12 and its volume is 314 cubic metre. Find its perpendicular height and slant height ($\pi = 3.14$)
4. Find the radius of a sphere if its volume is 904.32 cubic cm. ($\pi = 3.14$)
5. Total surface area of a cube is 864 sq.cm. Find its volume.
6. Find the volume of a sphere, if its surface area is 154 sq.cm.
7. Total surface area of a cone is 616 sq.cm. If the slant height of the cone is three times the radius of its base, find its slant height.
8. The inner diameter of a well is 4.20 metre and its depth is 10 metre. Find the inner surface area of the well. Find the cost of plastering it from inside at the rate Rs.52 per sq.m.
9. The length of a road roller is 2.1m and its diameter is 1.4m. For levelling a ground 500 rotations of the road roller were required. How much area of ground was levelled by the road roller? Find the cost of levelling at the rate of Rs. 7 per sq. m.

Answers

1. Basic Concepts in Geometry

Practice set 1.1

1. (i) 3 (ii) 3 (iii) 7 (iv) 1
(v) 3 (vi) 5 (vii) 2 (viii) 7

2. (i) 6 (ii) 8 (iii) 10 (iv) 1 (v) 3 (vi) 12

3. (i) P-R-Q (ii) Non collinear (iii) A-C-B (iv) Non collinear
(v) X-Y-Z (vi) Non collinear

4. 18 and 2 **5.** 25 and 9 **6.** (i) 4.5 (ii) 6.2 (iii) $2\sqrt{7}$ **7.** Triangle

Practice set 1.2

1. (i) No (ii) No (iii) Yes **2.** 4 **3.** 5 **4.** BP < AP < AB

5. (i) Ray RS or Ray RT (ii) Ray PQ (iii) Seg QR (iv) Ray QR and Ray RQ etc.
(v) Ray RQ and Ray RT etc.. (vi) Ray SR , Ray ST etc.. (vii) Point S

6. (i) Point A & Point C, Point D & Point P (ii) Point L & Point U, Point P & Point R
(iii) d (U,V) = 10, d (P,C) = 6 , d (V,B) = 3 , d (U,L) = 2

Practice set 1.3

1. (i) If a quadrilateral is a parallelogram then opposite angles of that quadrilateral are congruent.
(ii) If quadrilateral is a rectangle then diagonals are congruent.
(iii) If a triangle is an isosceles then segment joining vertex of a triangle and mid point of the base is perpendicular to the base

2. (i) If alternate angles made by two lines and its transversal are congruent then the lines are parallel.
(ii) If two parallel lines are intersected by a transversal the interior angles so formal are supplementary.
(iii) If the diagonals of a quaddrilateral are congruent then that quadrilateral is rectangle.

Problem set 1

1. (i) A (ii) C (iii) C (iv) C (v) B

2. (i) False (ii) False (iii) True (iv) False

3. (i) 3 (ii) 8 (iii) 9 (iv) 2 (v) 6 (vi) 22 (vii) 165

4. –15 and 1 **5.** (i) 10.5 (ii) 9.1 **6.** –6 and 8

2. Parallel Lines

Practice set 2.1

1. (i) 95° (ii) 95° (iii) 85° (iv) 85°

2. $\angle a = 70°$, $\angle b = 70°$, $\angle c = 115°$, $\angle d = 65°$

3. $\angle a = 135°$, $\angle b = 135°$, $\angle c = 135°$

5. (i) 75° (ii) 75° (iii) 105° (iv) 75°

Practice set 2.2

1. No. **4.** $\angle ABC = 130°$

Problem set 2

1. (i) C (ii) C (iii) A (iv) B (v) C **4.** $x = 130°$ $y = 50°$

5. $x = 126^{0}°$ **6.** f = 100° $g = 80°$

3. Triangles

Practice set 3.1

1. 110° **2.** 45° **3.** 80°, 60°, 40° **4.** 30°, 60°, 90°

5. 60°, 80°, 40° **6.** $\angle DRE = 70°$, $\angle ARE = 110°$

7. $\angle AOB = 125°$ **9.** 30°, 70°, 80°

Practice set 3.2

1. (i) SSC Test (ii) SAS Test (iii) ASA Test (iv) Hypotenuse Side Test.

2. (i) ASA Test, $\angle BAC \cong \angle QPR$,side AB ≅ side PQ, side AC ≅ side PR
(ii) SAS Test, $\angle TPQ \cong \angle TSR$, $\angle TQP \cong \angle TRS$, side PQ ≅ side SR

3. Hypotenuse Side Test, $\angle ACB \cong \angle QRP$, $\angle ABC \cong \angle QPR$, side AC ≅ side QR

4. SSS Test, $\angle MLN \cong \angle MPN$, $\angle LMN \cong \angle MNP$, $\angle LNM \cong \angle PMN$

Practice set 3.3

1. $x = 50°$, $y = 60°$, $m\angle ABD = 110°$, $m\angle ACD = 110°$.

2. 7.5 Units **3.** 6.5 Units **4.** l(PG) = 5 cm , l(PT) = 7.5 cm

Practice set 3.4

1. 2 cm **2.** 28° **3.** $\angle QPR$, $\angle PQR$ **4.** greatest side NA, smallest side FN

Practice set 3.5

1. $\frac{XY}{LM} = \frac{YZ}{MN} = \frac{XZ}{LN}$, $\angle X \cong \angle L$, $\angle Y \cong \angle M$, $\angle Z \cong \angle N$

2. l(QR) = 12 cm, l(PR) = 10 cm

Problem set 3

1. (i) D (ii) B (iii) B

5. Quadrilaterals

Practice set 5.1

1. $m\angle XWZ = 135^\circ$, $m\angle YZW = 45^\circ$, $l(WY) = 10$ cm

2. $x = 40^\circ$, $\angle C = 132^\circ$, $\angle D = 48^\circ$

3. 25 cm, 50 cm, 25 cm, 50 cm

4. 60°, 120°, 60°, 120°

6. $\angle A = 70^\circ$, $\angle B = 110^\circ$, $\angle C = 70^\circ$, $\angle R = 110^\circ$

Practice set 5.3

1. BO = 4 cm, $\angle ACB = 35^\circ$

2. QR = 7.5 cm, $\angle PQR = 105^\circ$, $\angle SRQ = 75^\circ$

3. $\angle IMJ = 90^\circ$, $\angle JIK = 45^\circ$, $\angle LJK = 45^\circ$

4. side = 14.5 cm, Perimetere = 58 cm

5. (i) False (ii) False (iii) True (iv) True (v) True (vi) False

Practice set 5.4

1. $\angle J = 127^\circ$, $\angle L = 72^\circ$ **2.** $\angle B = 108^\circ$, $\angle D = 72^\circ$

Practice set 5.5

1. XY = 4.5 cm,, YZ = 2.5 cm,, XZ = 5.5 cm

Problem set 5

1. (i) D (ii) C (iii) D **2.** 25 cm, **3.** $6.5\sqrt{2}$ cm

4. 24 cm, 32 cm, 24 cm, 32 cm **5.** PQ = 26 cm **6.** $\angle MPS = 65^\circ$

6. Circle

Practice set 6.1

1. 20 cm **2.** 5 cm **3.** 32 unit **4.** 9 unit

Practice set 6.2

1. 12 cm **2.** 24 cm

Problem set 6

1. (i) A (ii) C (iii) A (iv) B (v) D (vi) C (vii) D or B **2.** 2:1 **4.** 24 units

7. Co - ordinate Geometry

Practice set 7.1

1. point A : Quadrant II, point B : Quadrant III, point K : Quadrant I, point D : Quadrant I point E : Quadrant I, point F : Quadrant IV, point G : Quadrant IV, point H : Y-Axis. point M : X-Axis, point N : Y-Axis, point P : Y-Axis, point Q : Quadrant III
2. (i) Quadrant I (ii) Quadrant III (iii) Quadrant IV (iv) Quadrant II

Practice set 7.2

1. Square **2.** $x = -7$ **3.** $y = -5$ **4.** $x = -3$ **5.** 4
6. (i) Y-Axis, (ii) X-axis, (iii) Y-axis, (iv) X-axis,
7. To X-axis (5,0) , To Y-axis (0,5)
8. (-4,1), (-1.5, 1), (-1.5,5), (-4,5)

Problem set 7

1. (i) C (ii) A (iii) B (iv) C (v) C (vi) B
2. (i) Q (– 2,2), R(4,–1) (ii) T(0,–1), M(3,0) (iii) point S (iv) point O
3. (i) Quadrant IV (ii) Quadrant III
   (iii) Quadrant II (iv) Quadrant II (v) Y-axis (vi) X-axis
5. (i) 3 (ii) P(3,2), Q(3,-1), R (3,0) (iii) 0 **6.** . $y = 5, y = -5$ **7.** $|a|$

8. Trigonometry

Practice set 8.1

1. (i) $\frac{QR}{PQ}$ (ii) $\frac{QR}{PQ}$ (iii) $\frac{QR}{PR}$ (iv) $\frac{PR}{QR}$
2. (i) $\frac{a}{c}$ (ii) $\frac{b}{a}$ (iii) $\frac{b}{c}$ (iv) $\frac{a}{b}$
3. (i) $\frac{MN}{LN}$ (ii) $\frac{LM}{LN}$ (iii) $\frac{LM}{MN}$ (iv) $\frac{MN}{LN}$
4. (i) $\frac{PQ}{PR}, \frac{RQ}{PR}, \frac{PQ}{RQ}$ (ii) $\frac{QS}{PS}, \frac{PQ}{PS}, \frac{QS}{PQ}$

Practice set 8.2

1. $\sin\theta$: $\frac{12}{37}, \frac{1}{\sqrt{2}}, \frac{\sqrt{2}}{\sqrt{3}}, \frac{21}{29}, \frac{8}{17}, \frac{1}{3}$; $\cos\theta$: $\frac{60}{61}, \frac{1}{\sqrt{2}}, \frac{\sqrt{3}}{2}, \frac{20}{29}, \frac{15}{17}, \frac{4}{5}, \frac{2\sqrt{2}}{3}$

   $\tan\theta$: $\frac{12}{35}, \frac{11}{60}, \frac{1}{\sqrt{3}}, \sqrt{2}, \frac{3}{4}$

2. (i) $\frac{11}{2}$ (ii) $\frac{93}{20}$ (iii) 5 (iv) $\frac{2\sqrt{3}}{\sqrt{3}+1}$ (v) $\frac{3}{4}$ (vi) $\frac{\sqrt{3}}{2}$ **3.** $\frac{3}{5}$ **4.** $\frac{8}{17}$

Problem set 8

1. (i) A (ii) D (iii) C (iv) D

2. $\sin T = \frac{12}{13}$, $\cos T = \frac{5}{13}$, $\tan T = \frac{12}{5}$, $\sin U = \frac{5}{13}$, $\cos U = \frac{12}{13}$, $\tan U = \frac{5}{12}$

3. $\sin Y = \frac{8}{17}$, $\cos Y = \frac{15}{17}$, $\tan Y = \frac{8}{15}$, $\sin Z = \frac{15}{17}$, $\cos Z = \frac{8}{17}$, $\tan Z = \frac{15}{8}$

4. $\sin \theta = \frac{7}{25}$, $\tan \theta = \frac{7}{24}$, $\sin^2 \theta = \frac{49}{625}$, $\cos^2 \theta = \frac{576}{625}$

5. (i) 70 (ii) 60 (iii) 50

9. Surface Area and Volume

Practice set 9.1

1. 640 sq.cm, 1120 sq.cm. **2.** 20 Unit **3.** 81 sq.cm, 121.50 sq.cm.

4. 3600 sq.cm. **5.** 20 m **6.** 421.88 cubic cm

7. 1632.80 sq.cm, 4144.80 sq.cm. **8.** 21 cm

Practice set 9.2

1. 5 cm **2.** 36960 cubic cm. **3.** 10 cm, 6 cm **4.** ₹ 2640

5. 15 cm **6.** 8 cm **7.** 550 sq.cm **8.** 2816 sq.cm, 9856 cubic cm

9. 600 cubic metre **10.** 28.51 cubic metre, 47.18 sq.m.

Practice Set 9.3

1. (i) 200.96 sq.cm, 267.95 cubic cm. (ii) 1017.36 sq.cm, 3052.08 cubic cm.

(iii) 153.86 sq.m, 179.50 cubic cm.

2. 157 sq.cm, 235.5 sq.cm. **3.** 14130 cubic cm. **4.** 5544 sq.cm. **5.** 60 cm

Problem set 9

1. 1980 sq.m. **2.** 96801.6 cubic cm. **3.** 12 m, 13 m

4. 6 cm **5.** 1728 cubic cm. **6.** 179.67 cubic cm.

7. 21 cm **8.** 132 sq.m., ₹ 6864 **9.** 4620 sq.m, ₹ 32340

Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune.

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